10. [10 points] The following parts are unrelated.

a. [3 points] Consider the region in the first quadrant bounded by $y = -(x-2)(x-7)$ and the x-axis. Which of the following expressions represents the **perimeter** of the region?
Circle **one** option below.

i. $\displaystyle\int_2^7 -(x-2)(x-7)\,dx$

ii. $\displaystyle 5 + \int \sqrt{1+\left(-(x-2)(x-7)\right)^2}\,dx$

iii. $\displaystyle 5 + \int_2^7 \sqrt{1+(9-2x)^2}\,dx$

iv. $\displaystyle\int_2^7 \sqrt{1+(9-2x)^2}\,dx$

v. $\displaystyle\frac{1}{5}\int_2^7 -(x-2)(x-7)\,dx$

vi. NONE OF THESE

b. [4 points] Suppose that $f(x)$ is a differentiable function with a second derivative which is always positive. Suppose that $\text{LEFT}(40), \text{RIGHT}(40), \text{TRAP}(40)$, and $\text{MID}(40)$ are estimates of the integral $\displaystyle\int_0^{10} f(x)\,dx$. Which of the following **could** be true?
Circle **all** options which apply.

i. $\text{LEFT}(40) < \displaystyle\int_0^{10} f(x)\,dx < \text{TRAP}(40)$

ii. $\text{TRAP}(40) < \displaystyle\int_0^{10} f(x)\,dx < \text{RIGHT}(40)$

iii. $\displaystyle\int_0^{10} f(x)\,dx < \text{TRAP}(40) < \text{LEFT}(40) < \text{RIGHT}(40)$

iv. $\text{MID}(40) < \displaystyle\int_0^{10} f(x)\,dx < \text{LEFT}(40) = \text{RIGHT}(40)$

v. $\text{TRAP}(40) = 180$ and the average value of $f(x)$ on the interval $[0, 10]$ is 20.

vi. $\text{TRAP}(40) = 220$ and the average value of $f(x)$ on the interval $[0, 10]$ is 20.

vii. NONE OF THESE

c. [3 points] Birds gather in a large area centered around Marcy's bird bath. The population density of birds, measured in birds per square kilometer, at a radial distance r kilometers from the center of Marcy's bird bath is given by the function $p(r)$.
Which of the following expressions must represent the total number of birds found within 5 kilometers of the center of the bird bath? Circle **one** option below.

i. $25\pi p(r)$

ii. $\displaystyle 25\pi \int_0^5 p(r)\,dr$

iii. $\displaystyle\int_0^5 p(r)\,dr$

iv. $\displaystyle\int_{-5}^5 p(r)\,dr$

v. $\displaystyle\int_0^5 2\pi r p(r)\,dr$

vi. $\displaystyle\int_{-5}^5 2\pi r p(r)\,dr$

vii. $\displaystyle\int_0^5 \pi(p(r))^2\,dr$

viii. NONE OF THESE

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