

3. [15 points] Let $f(x)$ be a differentiable function whose derivative $f'(x)$ is always positive, and is also differentiable. Some values of $f(x)$ are given in the table below:

x	1	2	3	4	5	6
$f(x)$	1	3	6	13	20	22

Compute the exact value of the following integrals. If there is not enough information provided to determine the value of the integral, write “NEI” and clearly indicate why. Show all of your work.

a. [5 points] $\int_1^2 f'(3x)e^{f(3x)+3} dx.$

Solution: We use the substitution,

$$u = f(3x) + 3 \quad du = 3f'(3x)$$

Then

$$\int_1^2 f'(3x)e^{f(3x)+3} dx = \frac{1}{3} \int_9^{25} e^u du = \frac{1}{3} (e^{25} - e^9)$$

Answer: $\frac{1}{3} (e^{25} - e^9)$

b. [5 points] $\int_1^6 f'(x)(1 + \ln(f(x))) dx.$

Solution: We set

$$u = f(x),$$

$$du = f'(x)dx$$

$$v = 1 + \ln(f(x)),$$

$$dv = \frac{f'(x)}{f(x)} dx$$

Then, integrating by parts,

$$\begin{aligned} \int_1^6 f'(x)(1 + \ln(f(x))) dx &= f(x)(1 + \ln(f(x))) \Big|_1^6 - \int_1^6 f'(x) dx \\ &= f(x)(1 + \ln(f(x))) \Big|_1^6 - f(x) \Big|_1^6 \\ &= f(x) \ln(f(x)) \Big|_1^6 \\ &= 22 \ln(22) - 1 \ln(1) \\ &= 22 \ln(22) \end{aligned}$$

Answer: $22 \ln(22)$

c. [5 points] $\int_2^5 \frac{f'(x) + \frac{1}{x}}{(f(x) + \ln(x))^2} dx.$

Solution: Let

$$u = f(x) + \ln(x) \quad du = \left(f'(x) + \frac{1}{x} \right) dx$$

Then

$$\begin{aligned} \int_2^5 \frac{f'(x) + \frac{1}{x}}{(f(x) + \ln(x))^2} dx &= \int_{3+\ln(2)}^{20+\ln(5)} \frac{1}{u^2} du \\ &= -\frac{1}{u} \Big|_{3+\ln(2)}^{20+\ln(5)} \\ &= -\frac{1}{20 + \ln(5)} + \frac{1}{3 + \ln(2)} \end{aligned}$$

Answer: $\underline{-\frac{1}{20 + \ln(5)} + \frac{1}{3 + \ln(2)}}$