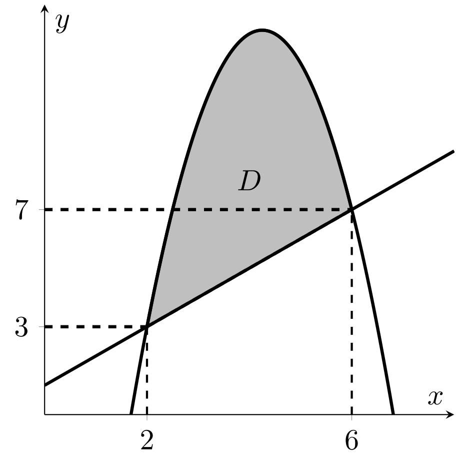
5. [15 points]

Katydyd Delemma owns a donut shop. She likes to experiment with different shapes for her donuts. Katydyd decides that she'd like to make new donuts using the region D, which is the region in the first quadrant bounded by the curves $y = x + 1$ and $y = -2x^2 + 17x - 23$, as shaded in the figure to the right. The two curves intersect at the points $(2, 3)$ and $(6, 7)$.



a. [5 points] Write an expression involving one or more integrals for the volume of the donut generated by revolving the region D about the y-axis. **Do not** evaluate any integrals in your expression.

Solution: We use vertical slices, which gives rise to shells. For this region, x ranges between 2 and 6, so we get

$$\int_2^6 2\pi x \left((-2x^2 + 17x - 23) - (x+1)\right)\, dx = \int_2^6 2\pi x \left(-2x^2 + 16x - 24\right)\, dx$$
$$= -4\pi \int_2^6 x\,(x-2)\,(x-6)\, dx$$

Answer: $\int_2^6 2\pi x \left(-2x^2 + 16x - 24\right)\, dx$

b. [5 points] Write another expression involving one or more integrals for the volume of the solid generated by revolving the region about the line $y = -1$. **Do not** evaluate any integrals in your expression.

Solution: We use vertical slices, which gives rise to washers. For this region, x ranges between 2 and 6, so we get

$$\int_2^6 \pi \left((-2x^2 + 17x - 23 - (-1))^2 - (x + 1 - (-1))^2\right)\, dx$$
$$= \int_2^6 \pi \left((-2x^2 + 17x - 22)^2 - (x+2)^2\right)\, dx$$
$$= \int_2^6 \pi \left(4x^4 - 68x^3 + 376x^2 - 752x + 480\right)\, dx$$

Answer: $\int_2^6 \pi \left((-2x^2 + 17x - 22)^2 - (x+2)^2\right)\, dx$

c. [5 points] Katydyd decides that she doesn't want to deal with all the oddly-shaped donut holes, so she decides to make little cakes instead. She wants the base to be the region D pictured above, with square cross-sections perpendicular to the x-axis. Find an expression involving one or more integrals for the volume of Katydyd's cakes. **Do not** evaluate any integrals in your expression.

Solution: The volume of a slice of small width Δx is approximately

$$((-2x^2+17x-23)-(x+1))^2\Delta x = (-2x^2+16x-24)\Delta x$$

Since x ranges between 2 and 6 in this region, the total volume of one cake is

$$\int_2^6 ((-2x^2+17x-23)-(x+1))^2\,dx = \int_2^6 (-2x^2+16x-24)^2\,dx$$
$$= \int_2^6 4\left((x-2)(x-6)\right)^2\,dx$$

Answer: $\int_2^6 (-2x^2+16x-24)^2\,dx$