6. [7 points] Use the partial fraction decomposition

$$\frac{x^2+13x+10}{(7-x)(x^2+1)} = \frac{3}{7-x} + \frac{2x+1}{x^2+1}$$

to evaluate the following indefinite integral, showing all of your work.

$$\int \frac{x^2+13x+10}{(7-x)(x^2+1)}\,dx$$

Solution:

$$\begin{aligned}\int \frac{x^2+13x+10}{(7-x)(x^2+1)}\,dx &= \int \frac{3}{7-x} + \frac{2x+1}{x^2+1}\,dx \\ &= \int \frac{3}{7-x}\,dx + \int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx \\ &= -3\ln|7-x| + \ln|x^2+1| + \arctan x + C\end{aligned}$$

Answer: $-3\ln|7-x| + \ln|x^2+1| + \arctan x + C$

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