7. [10 points] On her latest hiking expedition, Emily has scaled up a cliff face with a height of 30 feet, and is now using her rope to pull up her camping supplies after her. The supplies are initially on the ground, and are pulled directly upward by the rope. Assume that the rope initially extends from the top of the cliff to the ground in a straight vertical line (so that the rope initially has length 30 feet). The rope has a density of 0.5 pounds per foot, and the supplies weigh 35 pounds.

a. [2 points] Suppose that Emily has lifted the supplies x feet from the ground (so that Emily has reeled in x feet of the rope). Find an expression in terms of x for the total weight, in pounds, of the supplies and the remaining length of rope being used to support the supplies.

Solution: The supplies weigh 35 pounds, and each foot of rope weighs 0.5 pounds. After the supplies have been lifted x feet, the total length of rope is $30 - x$ feet, and so the total weight, in pounds, is $35 + 0.5(30 - x)$.

Answer: $35 + 0.5(30 - x) = 50 - 0.5x$.

b. [4 points] Find an expression involving one or more integrals for the total work done in lifting the supplies 20 feet above the ground using the rope. Do **not** evaluate any integrals in your expression. Include units.

Solution: Using our expression from part a, we see that if the supplies have been lifted x feet already, then the work, in foot-pounds, done to raise the supplies a short extra distance Δx feet would be

$$(50 - 0.5x)\Delta x.$$

so the total work done in lifting the supplies from 0 to 20 feet above the ground is

$$\int_0^{20} (50 - 0.5x)\, dx.$$

Answer: $\int_0^{20} (50 - 0.5x)\, dx$

Answer: Units: **ft·lbs**

c. [4 points] After lifting the supplies 20 feet, the water tank in Emily's supplies bag is pierced, and begins to leak, so that the weight of the supplies decreases by 0.2 pounds per foot. Find an expression involving one or more integrals for the total work done in lifting the supplies the remaining distance of 10 feet using the rope. Do **not** evaluate any integrals in your expression. Include units.

Solution: We will present two different solution methods:
First, using our expression from part a. combined with the fact that the weight of the supplies decreases by 0.2 pounds for every foot it is raised above 20 feet, we see that the combined weight of the rope and supplies, for $20 \leq x \leq 30$ is

$$50 - 0.5x - 0.2(x - 20) = 54 - 0.7x.$$

Therefore, the work done to move a small extra distance Δx feet is

$$(54 - 0.7x)\Delta x,$$

and so the total work done is

$$\int_{20}^{30} (54 - 0.7x)\, dx.$$

Alternatively, the weight of the rope plus bag at height $20 + r$ is

$$\begin{aligned} \text{weight of supplies} + \text{weight of rope} &= (35 - 0.2r) + (30 - (20 + r))(0.5) \\ &= 40 - 0.7r \end{aligned}$$

So the work required to move an extra Δr feet would be

$$(40 - 0.7r)\Delta r$$

and r ranges from 0 to 10 feet, so the total work done is

$$\int_0^{10} 40 - 0.7r\, dr$$

Answer: $\int_{20}^{30} (54 - 0.7x)\, dx = \int_0^{10} 40 - 0.7r\, dr$

Answer: Units: **ft·lbs**