8. [8 points] Some of the values of a continuous, concave down, increasing function $g(x)$ are given in the following table:

x	0	1	2	3	4	5	6
$g(x)$	-17	-4	4	9	12	14	15

a. [3 points] Find the MID(3) approximation to

$$\int_0^6 g(x)\,dx.$$

Write out all the terms in your sum. You do not need to simplify, but your final answer should not involve the letter g.

Solution: The interval $[0,6]$ has width 6, so we should divide it into three subintervals of width 2. This means we should plug $x=1$, $x=3$, and $x=5$ into the function $g(x)$. We obtain:

$$\begin{aligned}\text{MID}(2) &= 2\left(g(1)+g(3)+g(5)\right)\\ &= 2(-4+9+14)\\ &= 2(19)\\ &= 38.\end{aligned}$$

Answer: 38

b. [2 points] Is the MID(3) estimate to $\int_0^6 g(x)\,dx$ you found in part (a) an underestimate, an overestimate, or is there not enough information (NEI)? Circle your choice. No justification is required.

Circle one: UNDERESTIMATE **OVERESTIMATE** NEI

c. [3 points] Find the RIGHT(3) approximation to

$$\int_0^6 3xg\left(\frac{x}{2}\right)\,dx.$$

Write out all the terms in your sum. You do not need to simplify, but your final answer should not involve the letter g.

Solution: The interval $[0,6]$ has width 3, so we should divide it into three subintervals of width 2. This means we should plug $x=2$, $x=4$, and $x=6$ into the function $3xg\left(\frac{x}{2}\right)$. We obtain:

$$\begin{aligned}\text{RIGHT}(3) &= 2\left(3(2)g(1)+3(4)g(2)+3(6)g(3)\right)\\ &= 2(6(-4)+12(4)+18(9))\\ &= 2(-24+48+162)\\ &= 2(186) = 372.\end{aligned}$$

Answer: 372

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