1. [11 points] Let $f(x)$ be a **twice-differentiable** function defined for all real numbers. Suppose that f also satisfies the following:

$$f(1) = -2, \qquad f(3) = 4, \qquad f(11) = 7,$$

$$\int_1^3 f(x)\,dx = 5, \qquad \int_3^{11} f(x)\,dx = 14$$

Compute the exact value of the following quantities. If there is not enough information provided to answer the question, write "NEI" and clearly indicate why. Show all of your work.

a. [3 points] The average value of $f(x)$ on the interval $[1, 11]$

Solution: The average value is

$$\frac{1}{11-1}\int_1^{11} f(x)\,dx = \frac{1}{10}\left(\int_1^3 f(x)\,dx + \int_3^{11} f(x)\,dx\right) = \frac{1}{10}(5+14) = 1.9.$$

Answer: 1.9

b. [4 points] $\displaystyle\int_1^3 yf'(y)\,dy$

Solution: Integrating by parts, we get

$$\begin{aligned}\int_1^3 yf'(y)\,dy &= yf(y)\Big|_1^3 - \int_1^3 f(y)\,dy \\ &= 12 + 2 - 5 \\ &= 9.\end{aligned}$$

Answer: 9

c. [4 points] $\displaystyle\int_1^3 xf(x^2+2)\,dx$

Solution: Using the substitution $u = x^2 + 2$, and remembering to change the bounds, we see:

$$\int_1^3 xf(x^2+2)\,dx = \frac{1}{2}\int_3^{11} f(u)\,du = 7.$$

Answer: 7