3. [9 points] Consider the following function:

$$F(x) = 2 + \int_{-1}^{\sin(x)} \frac{1+t^2}{1+t^4}\,dt.$$

a. [2 points] Find a value of a such that $F(a) = 2$.

Solution: Since $\frac{1+t^2}{1+t^4} \geq 0$ for all t, we must have $F(x) \geq 2$. Note that if $\sin(a) = -1$, then we have

$$F(a) = 2 + \int_{-1}^{-1} \frac{1+t^2}{1+t^4}\,dt = 2 + 0 = 2.$$

So we want $\sin(a) = -1$. We choose $a = \frac{3\pi}{2}$, but

$$a = -\frac{\pi}{2} + 2k\pi$$

would work for any integer k.

Answer: $a =$ $3\pi/2$

b. [3 points] Calculate $F'(x)$.

Solution: We can write $F(x) = G(\sin(x))$ where

$$G(x) = 2 + \int_{-1}^{x} \frac{1+t^2}{1+t^4}\,dt$$

and we know from the Second Fundamental Theorem of Calculus that

$$G'(x) = \frac{1+x^2}{1+x^4}$$

Therefore, by the Chain Rule,

$$F'(x) = G'(\sin(x))(\sin(x))' = \cos(x)\frac{1+\sin^2(x)}{1+\sin^4(x)}.$$

Answer: $F'(x) =$ $\dfrac{\cos(x)(1+\sin^2(x))}{(1+\sin^4(x))}$

c. [4 points] Find a function $f(t)$ and constants a and C so that we may rewrite $F(x)$ in the form $\int_a^x f(t)\,dt + C$. There may be more than one correct answer.

Solution: Given the expression of $F'(x)$ we obtained in **b.** and that $F(3\pi/2) = 2$ as calculated in **a.**, we may write:

$$F(x) = 2 + \int_{3\pi/2}^{x} \frac{(1+\sin^2(t))\cos(t)}{1+\sin^4(t)}\,dt.$$

$f(t) =$ $\dfrac{(1+\sin^2(t))\cos(t)}{1+\sin^4(t)}$ $a =$ $\dfrac{3\pi}{2}$ $C =$ 2