

5. [12 points] Melissa is an ecologist who finds that a new and increasingly popular brand of pesticide is harming native fish populations. She begins an information campaign about the dangers of the pesticide, and tracks $p(t)$, the **rate of change** of the concentration of pesticides in a local lake t days after the start of the campaign. The concentration of pesticides is measured in micrograms per liter ($\mu g/L$) so that the function $p(t)$ is measured in $\mu g/L$ per day. Ten days after the start of the information campaign, Melissa finds that the concentration of pesticides in the lake is 360 micrograms per liter.

- a. [4 points] Find an expression involving one or more integrals for the concentration, in micrograms per liter, of pesticides in the lake at the start of the information campaign. Your answer will involve the letter p .

Answer: $360 - \int_0^{10} p(t) \, dt$

Melissa finds the following table of values for $p(t)$ and $p'(t)$. She also discovers that $p'(t)$ is always decreasing for all $t > 0$.

t	2	4	6	8	10
$p(t)$	1.5	3	2	-1	-6
$p'(t)$	2	0	-1	-2	-3

- b. [5 points] Melissa would like to find an **overestimate** to the integral

$$\int_2^{10} p(t) \, dt.$$

Which one of the following methods for approximating integrals would be guaranteed to give an overestimate?

Circle one: LEFT(n) RIGHT(n) MID(n) TRAP(n)

Find the approximation you chose above for the integral $\int_2^{10} p(t) \, dt$. You should use the maximal amount of equal subintervals possible, and write out all the terms in your sum.

Solution: The best we can do is MID(2), and the midpoints are 4, 8. Therefore, the approximation is $4p(4) + 4p(8) = 12 - 4 = 8$.

Answer: 8

- c. [3 points] Recall the information about $p'(t)$ given above part **b.**, and that 10 days after the start of the information campaign, the pesticide concentration is 360 micrograms per liter. Melissa estimates the integral $\int_{10}^{20} p(t) \, dt$ using RIGHT(15) and finds that the result is -250 . Which of the following statements **must** be true?

- i. 20 days after the start of the information campaign, the concentration of pesticides must be **greater than** 110 micrograms per liter.
- ii. 20 days after the start of the information campaign, the concentration of pesticides must be **less than** 110 micrograms per liter.
- iii. 20 days after the start of the information campaign, the concentration of pesticides must be **equal to** 110 micrograms per liter.
- iv. None of the above.