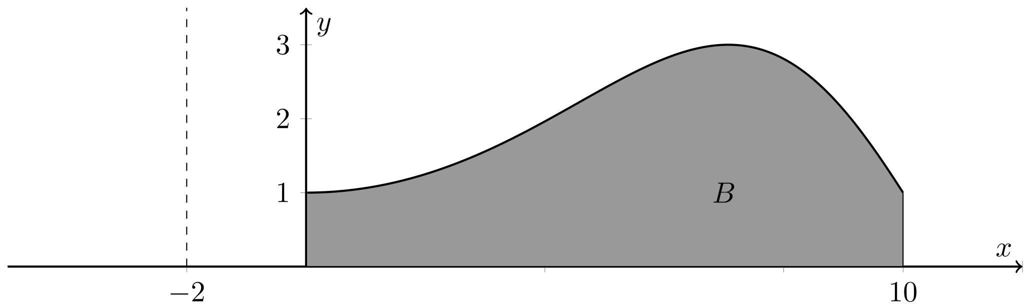
7. [9 points] Consider the region B in the first quadrant, bounded by $x = 10$ and

$$f(x) = 1 + 2\sin\left(\frac{\pi x^2}{100}\right), \quad 0 \le x \le 10$$

as shaded in the diagram below.



a. [4 points] Pianta designs a beautiful decorative urn by rotating the shaded region B around the x-axis. Write an expression involving one or more integrals for the volume of the urn. Your expression should not involve the letter f. **Do not** evaluate any integrals in your expression.

Solution: We can slice the solid into disks with tiny horizontal width. For a disk located at x, the radius of the base is

$$f(x) = 1 + 2\sin\left(\frac{\pi x^2}{100}\right).$$

so the solid has volume

$$\int_0^{10} \pi\left[1 + 2\sin\left(\frac{\pi x^2}{100}\right)\right]^2 dx.$$

Answer: $\displaystyle\int_0^{10} \pi\left[1 + 2\sin\left(\frac{\pi x^2}{100}\right)\right]^2 dx$

b. [5 points] Pianta also designs a fountain basin for a city plaza by rotating the shaded region B around the line $x = -2$ depicted above. Write an expression involving one or more integrals for the volume of the fountain basin. Your expression should not involve the letter f. **Do not** evaluate any integrals in your expression.

Solution: We can slice the region B into horizontal slices, which after rotation become thin cylindrical shells. The volume of a thin shell formed by rotating a slice located at x is given by

$$2\pi(2+x)f(x)\Delta x = 2\pi(2+x)\left[1 + 2\sin\left(\frac{\pi x^2}{100}\right)\right]\Delta x,$$

so the volume is given by

$$\int_0^{10} 2\pi(2+x)\left[1 + 2\sin\left(\frac{\pi x^2}{100}\right)\right] dx.$$

Answer: $\displaystyle\int_0^{10} 2\pi(2+x)\left[1 + 2\sin\left(\frac{\pi x^2}{100}\right)\right] dx$