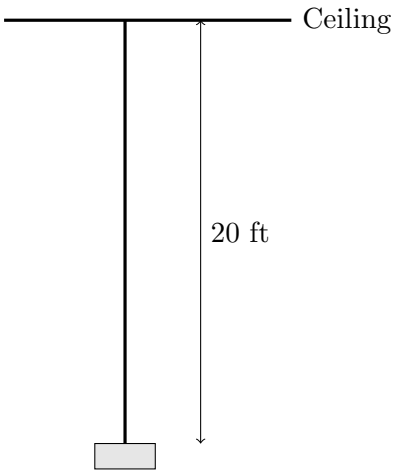
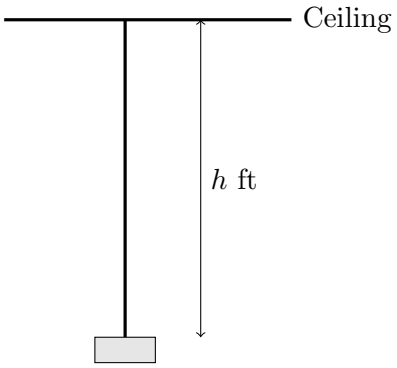
8. [8 points] A theater technician is raising a pendant lamp. A chain of length 20 feet hangs from the ceiling and a lamp weighing 5 lbs is attached to the bottom of the chain, as depicted below to the left. The density of the chain is 0.6 lb/ft. The technician pulls the lamp and chain straight upward toward the ceiling. As the lamp is lifted, the technician does not need to lift the portion of the chain that has already been "reeled in", that is, the part that has reached the ceiling. The lamp is lifted in this manner until it is 12 feet above its initial position.





a. [4 points] Suppose the lamp has been lifted until it is h feet from the ceiling, as shown in the diagram on the top right. Write an expression that approximates the work required to lift the chain and the lamp by a small distance Δh feet. Your answer should not involve any integrals, and should be expressed in terms of h and Δh. Include units.

Solution: The weight of the rope plus the lamp is given by

$$0.6h + 5 \quad \text{ft} \cdot \text{lb},$$

so the work done is

$$(0.6h + 5)\Delta h \quad \text{ftlbs}.$$

Answer: $(0.6h + 5)\Delta h$ **Units:** ftlbs

b. [4 points] Write an expression involving one or more integrals that represents the total work required to raise the lamp 12 feet above its initial position using the chain. **Do not** evaluate any integrals in your expression. Include units.

Solution: The lamp is initially 20 ft from the ceiling, and when the lamp is at its final position, it is 8 ft below the ceiling. Therefore, the total work done is

$$\int_8^{20} (0.6h + 5)\, dh \quad \text{ftlbs}.$$

Answer: $\displaystyle\int_8^{20} (0.6h + 5)\, dh$ **Units:** ftlbs