9. [12 points]

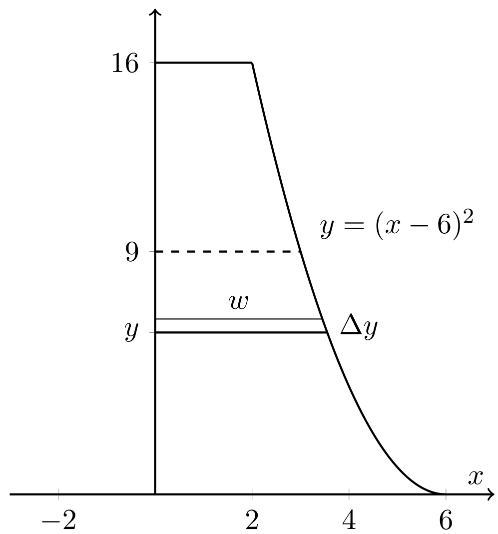
A big tank at a chemical factory is formed by rotating the region in the first quadrant bounded by $y = 16$, and

$$y = (x-6)^2,$$

around the y-axis. All distances are measured in meters. The tank is filled with liquid chemicals up to $y = 9$ meters, as shown by the dashed line in the plot to the right. Due to sedimentation, the liquid has a varying density of

$$f(y) = 3 - 0.1y \quad \text{kg/m}^3$$

at height y. Workers at the factory will pump the chemicals out through the top of the tank. You may assume that the acceleration due to gravity is $g = 9.8\text{m/s}^2$.



a. [2 points] Consider the thin horizontal strip of the region depicted above, which is located y meters above the x-axis. It has horizontal length w and a small thickness Δy. Find a formula for w in terms of y.

Solution: We know that $y = (w-6)^2$ and $w \le 6$, so $w = 6 - \sqrt{y}$.

Answer: $w =$ $6 - \sqrt{y}$

b. [4 points] When the strip above is rotated around the y-axis, it forms a thin **disk**. Write an expression which approximates the **mass** of that disk. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Solution: The volume of a slice is $\pi w^2 \Delta y$. To find the mass of a slice, we must multiply by the density, giving us $\pi w^2 f(y)\Delta y = \pi(6-\sqrt{y})^2(3-0.1y)\Delta y$.

Answer: $\pi(6-\sqrt{y})^2(3-0.1y)\Delta y$ **Units:** kg

c. [3 points] Write an expression which approximates the work needed to lift the thin disk described in part **b** to the top of the tank. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Solution: We multiply the mass of the slice by g to get its weight, and then multiply by the distance it travels to get the work done on the slice. Therefore, the work done is $\pi(6-\sqrt{y})^2(3-0.1y)(9.8)(16-y)\Delta y$.

Answer: $9.8\pi(6-\sqrt{y})^2(3-0.1y)(16-y)\Delta y$ **Units:** Joules

d. [3 points] Write an expression involving one or more integrals representing the work needed to pump all the liquid chemicals to top of the tank, using the same units as in part **c**. **Do not** evaluate any integrals in your expression.

Solution: We note that the lower bound of y should be 0, and the upper bound of y should be 9. Thus the total work done is $\int_0^9 9.8\pi(6-\sqrt{y})^2(3-0.1y)(16-y)\,dy$ Joules.

Answer: $\int_0^9 9.8\pi(6-\sqrt{y})^2(3-0.1y)(16-y)\,dy$