

1. (3 pts each) Circle true or false. No explanation necessary.

(a) If $0 < f(x) < g(x)$ for all x , then $\int_1^4 \frac{g(x)}{f(x)} > 3$.

True $\int_1^4 \frac{g(x)}{f(x)} dx > \int_1^4 \frac{f(x)}{f(x)} dx = \int_1^4 1 dx = x \Big|_1^4 = 4 - 1 = 3.$

(b) $\int_{\pi}^{\pi} \frac{\sin^2(x^5) - \cos(42x)}{x^2 + x + 1} > 2.$

False The integral is from π to π , so it's 0.

(c) $\int_1^{\infty} \frac{1}{x^{71}} dx$ converges.

True $\int_1^{\infty} \frac{1}{x^{71}} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-71} dx = \lim_{b \rightarrow \infty} -\frac{1}{70} x^{-70} \Big|_1^b = \lim_{b \rightarrow \infty} -\frac{1}{70b^{70}} + \frac{1}{70} = \frac{1}{70}.$

(d) If $g(x) + 2$ is an antiderivative of $f(x)$, then $g(x)$ is also an antiderivative of $f(x)$.

True $g(x) + 2$ and $g(x)$ have the same derivative.

2. (6 pts) State the construction theorem for antiderivatives.

If $f(x)$ is continuous on the closed interval $[a, b]$, then the function $F(x) = \int_a^x f(t) dt$ is an antiderivative of $f(x)$ for $a \leq x \leq b$.