1. (3 pts each) Circle true or false. No explanation necessary.

(a) If $0 < f(x) < g(x)$ for all $x$, then $\int_1^4 \frac{g(x)}{f(x)} > 3$.

True

$$\int_1^4 \frac{g(x)}{f(x)} \, dx > \int_1^4 \frac{f(x)}{f(x)} \, dx = \int_1^4 1 \, dx = x|_1^4 = 4 - 1 = 3.$$ 

(b) $\int_{\pi}^{\pi} \frac{\sin^2(x^5) - \cos(42x)}{x^2 + x + 1} > 2$.

False

The integral is from $\pi$ to $\pi$, so it’s 0.

(c) $\int_1^{\infty} \frac{1}{x^7} \, dx$ converges.

True

$$\int_1^{\infty} \frac{1}{x^7} \, dx = \lim_{b \to \infty} \int_1^b x^{-7} \, dx = \lim_{b \to \infty} -\frac{1}{7} x^{-7} \bigg|_1^b = \lim_{b \to \infty} -\frac{1}{7} b^{-7} + \frac{1}{7} = \frac{1}{7}.$$ 

(d) If $g(x) + 2$ is an antiderivative of $f(x)$, then $g(x)$ is also an antiderivative of $f(x)$.

True

$g(x) + 2$ and $g(x)$ have the same derivative.

2. (6 pts) State the construction theorem for antiderivatives.

If $f(x)$ is continuous on the closed interval $[a, b]$, then the function $F(x) = \int_a^x f(t) \, dt$ is an antiderivative of $f(x)$ for $a \leq x \leq b$. 

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Winter, 2003 Math 116 Exam 1 Problem 2 Solution