4. (6 pts) Write an integration problem (of your choice) for which the substitution \( w = 1/x \) would be the best way to start. You need not evaluate your own integral.

Many choices here. A good one will have \( 1/x \) inside some other function, and its derivative (up to a constant) outside. So

\[
\int \frac{\sin(1/x)}{x^2} \, dx \quad \text{and} \quad \int 3x^{-2} \ln(x^{-1}) \, dx
\]

are good choices.

5. (10 pts) Does

\[
\int_0^8 \frac{5 + \sin(x)}{x(8 + \cos(x))} \, dx
\]

converge or diverge? Demonstrate unequivocally that your answer is correct.

We’ll use the comparison test to show that the integral diverges. Since \( \sin x \) is between \(-1\) and \(1\), \(5 + \sin x\) is between \(4\) and \(6\). Likewise since \(\cos x\) is between \(-1\) and \(1\), \(8 + \cos x\) is between \(7\) and \(9\). It follows that

\[
\frac{4}{9} \leq \frac{5 + \sin(x)}{8 + \cos(x)} \leq \frac{6}{7}
\]

for all values of \(x\). Therefore

\[
\int_0^8 \frac{5 + \sin(x)}{x(8 + \cos(x))} \, dx \geq \int_0^8 \frac{4}{9} \cdot \frac{1}{x} \, dx = \lim_{a \to 0^+} \frac{4}{9} \int_a^8 \frac{1}{x} \, dx = \lim_{a \to 0^+} \frac{4}{9} \ln x \bigg|_a^8 = \frac{4}{9} \lim_{a \to 0^+} \ln(8) - \ln(a).
\]

Since \(-\ln(a)\) approaches \(\infty\) as \(a\) approaches \(0\), the final expression diverges. So the original integral diverges by the comparison test.