4. (6 pts) Write an integration problem (of your choice) for which the substitution $w = 1/x$ would be the best way to start. You need not evaluate your own integral.

Many choices here. A good one will have $1/x$ inside some other function, and its derivative (up to a constant) outside. So

$$\int \frac{\sin(1/x)}{x^2}\,dx \quad \text{and} \quad \int 3x^{-2}\ln(x^{-1})\,dx$$

are good choices.

5. (10 pts) Does

$$\int_0^8 \frac{5+\sin(x)}{x(8+\cos(x))}\,dx$$

converge or diverge? Demonstrate unequivocally that your answer is correct.

We'll use the comparison test to show that the integral diverges. Since $\sin x$ is between -1 and 1, $5+\sin x$ is between 4 and 6. Likewise since $\cos x$ is between -1 and 1, $8+\cos x$ is between 7 and 9. It follows that

$$\frac{4}{9} \le \frac{5+\sin(x)}{8+\cos(x)} \le \frac{6}{7}$$

for all values of x. Therefore

$$\int_0^8 \frac{5+\sin(x)}{x(8+\cos(x))}\,dx \ge \int_0^8 \frac{4}{9}\cdot\frac{1}{x}\,dx = \lim_{a\to 0^+}\frac{4}{9}\int_a^8 \frac{1}{x}\,dx = \lim_{a\to 0^+}\frac{4}{9}\ln x\Big|_a^8 = \frac{4}{9}\lim_{a\to 0^+}\ln(8)-\ln(a).$$

Since $-\ln(a)$ approaches ∞ as a approaches 0, the final expression diverges. So the original integral diverges by the comparison test.