8. (10 pts) Calculate the exact value of this definite integral. You will be graded on the correctness of your work, so show it carefully.

\[ \int_{-\infty}^{0} \frac{e^x}{1 + e^x} \, dx \]

Let \( w = 1 + e^x \). Then \( dw = e^x \, dx \), so we have

\[ \int_{-\infty}^{0} \frac{e^x}{1 + e^x} \, dx = \int_{x = -\infty}^{x = 0} \frac{dw}{w} = \ln w \bigg|_{x = -\infty}^{x = 0} = \ln(1 + e^0) - \ln(1 + e^{-\infty}) \]

Since \( e^a \) approaches 0 as \( a \) approaches \( -\infty \), that’s

\[ \ln(1 + 1) - \ln(1 + 0) = \ln 2 \]