

8. (10 pts) Calculate the exact value of this definite integral. You will be graded on the correctness of your work, so show it carefully.

$$\int_{-\infty}^0 \frac{e^x}{1+e^x} dx$$

Let $w = 1 + e^x$. Then $dw = e^x dx$, so we have

$$\begin{aligned} \int_{-\infty}^0 \frac{e^x}{1+e^x} dx &= \int_{x=-\infty}^{x=0} \frac{dw}{w} = \ln w \Big|_{x=-\infty}^{x=0} = \ln(1+e^x) \Big|_{-\infty}^0 \\ &= \lim_{a \rightarrow -\infty} \ln(1+e^x) \Big|_a^0 = \lim_{a \rightarrow -\infty} \ln(1+e^0) - \ln(1+e^a). \end{aligned}$$

Since e^a approaches 0 as a approaches $-\infty$, that's

$$\ln(1+1) - \ln(1+0) = \boxed{\ln 2}.$$