

1. (14 pts.) Suppose that f and its derivative f' are continuous functions such that $f(0) = -1$, $f(2) = 3$, $f'(0) = 3$, $f'(2) = 4$, and $\int_0^2 f(x) dx = 1.5$. Compute each of the following definite or indefinite integrals. Be sure to show your work.

(a) $\int f'(x)e^{2f(x)} dx$

$$= \frac{1}{2}e^{2f(x)} + C$$

$$\text{because } \frac{d}{dx} \left\{ \frac{1}{2}e^{2f(x)} + C \right\} = \frac{1}{2}e^{2f(x)} \cdot 2f'(x) \text{ by the chain rule.}$$

(b) $\int_0^1 f(2x) dx$

$$= \int_0^2 f(w) \frac{dw}{2} \quad \text{by } w = 2x, dw = 2dx, x = 0, x = 1 \implies w = 0, w = 2$$

$$= \frac{1}{2} \int_0^2 f(w) dw = \frac{1.5}{2} = 0.75$$

(c) $\int_0^2 x f''(x) dx$

$$= x f'(x) \Big|_0^2 - \int_0^2 f'(x) dx \quad \text{using } u = x, du = dx, dv = f''(x) dx, v = f'(x)$$

$$= 2 \cdot 4 - 0 - 3 + (-1) = 8 - 4 = 4$$