3. (12 pts.) Some distance upriver from a small reservoir, there has been a chemical spill. The authorities are concerned with levels of the chemical in the reservoir. Consequently, they take samples at half hour intervals of the rate $r(t)$, in gallons per hour, that the chemical is entering the reservoir t hours after the chemical spill. Their data is recorded in the following table.

t	0	.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$r(t)$	0	0	0	0	0	0.175	.4	.675	1	1.375	1.8

(a) Write an integral that represents the total amount of chemical that has entered the reservoir during the first five hours after the spill.

$\int_0^5 r(t)\,dt$

(b) Based on the data given, find the left and right hand sum approximations to your integral. (Show how you computed the sums.)

LHS: $0.5(0+0+0+0+0+0.175+0.4+0.675+1+1.375) = 1.8125$
RHS: $0.5(0+0+0+0+0.175+0.4+0.675+1+1.375+1.8) = 2.7125$

LHS = 1.8125 gallons

RHS = 2.7125 gallons

(c) Is it reasonable to expect that LHS is a lower bound for the integral? Explain why or why not.

It is reasonable to expect that LHS is a lower bound for the integral because 1) we can see that $r(t)$ appears to be an increasing, concave-up function by calculating the difference quotients from the above table,
2) LHS is a lower bound for increasing functions

(d) What is your best estimate of the integral based on the given data? Do you think it would be an under- or over-estimate of the actual value of the integral? Explain the reason for your answer.

Since the left hand sum is an underestimate and the right hand sum is an over-estimate of the integral, their average, the trapezoidal approximation, will likely be a a better estimate of the integral. In general, we expect the trapezoidal estimate, in this case $TRAP = (LHS + RHS)/2 = 2.2625$, to give a better estimate because the lines given by taking the tops of the trapezoids usually give a better approximation to the function $r(t)$ than do the horizontal line seqments that are the tops of the rectangles in the left and right hand sums.
We expect the trapezoidal rule to be an overestimate since $r(t)$ appears to be concave up (r seems to be increasing at an increasing rate), so the tops of the trapezoids used to approximate the area under the graph of r lie above the graph.