6. (10 points) **(a)** Explain why $\displaystyle\int_0^{\pi/3} \frac{\cos x}{\sin^2 x}\,dx$ is an improper integral.

Because the integrand, $\displaystyle\frac{\cos(x)}{\sin^2(x)}$**, is unbounded for** x **near 0.**

(b) Show how the improper integral in part (a) is defined mathematicially.

$\displaystyle\int_0^{\pi/3} \frac{\cos x}{\sin^2 x}\,dx = \lim_{a\to 0+} \int_a^{\pi/3} \frac{\cos x}{\sin^2 x}\,dx$ **provided the limit is a finite number.**

Otherwise, the integral diverges.

(c) If the improper integral in part **(a)** converges, then use your definition to calculate the value to which it converges. If the improper integral in part **(a)** does not converge, then explain why not. Show your work.

$$\int_a^{\frac{\pi}{3}} \frac{\cos x}{\sin^2 x}\,dx = \int_{\sin(a)}^{\sin(\frac{\pi}{3})} \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_{\sin(a)}^{\sin(\frac{\pi}{3})} = -\frac{1}{\sin(\frac{\pi}{3})} + \frac{1}{\sin(a)}$$

by $u = \sin(x)$**,** $du = \cos(x)dx$**.**

$\displaystyle\lim_{a\to 0+} \int_a^{\frac{\pi}{3}} \frac{\cos x}{\sin^2 x}\,dx = -\frac{1}{\sin(\frac{\pi}{3})} + \lim_{a\to 0+} \frac{1}{\sin(a)}$ **and the last limit does not exist since** $\sin(a) \to 0$ **as** $a \to 0^+$**. Therefore, the integral diverges.**