7. (15 points) Let g be the function that is defined for $x > 1$ by

$$g(x) = \int_3^x \frac{t}{\ln t}\, dt.$$

(a) Find $g'(x)$.

$g'(x) = \dfrac{x}{\ln(x)}$

(b) On which subinterval(s) of $x > 1$, if any, is g increasing? Briefly explain the reason for your answer.

g is increasing where $g' > 0$, or where $\dfrac{x}{\ln(x)} > 0$. Thus g is increasing on every subinterval of $x > 1$.

(c) On which subinterval(s) of $x > 1$, if any, is g concave up? Briefly explain the reason for your answer.

g is concave up where g' is increasing or where $g'' > 0$. Now, $g''(x) = \dfrac{\ln(x) - 1}{(\ln(x))^2}$. Since the denominator is always positive, this expresssion is zero at $x = e$, positive for $x > e$ where $\ln x > 1$, and negative for $1 < x < e$ where $\ln x - 1 < 0$. Therefore, $g''(x) > 0$ and g is concave up for $x > e$.

(d) Fill in the blanks with one of the words "positive", "negative", or "zero", to make the following sentences true. (Any word may be used more than once. No explanation necessary.)

$g(4)$ is positive.

$g(3)$ is zero.

$g(2)$ is negative.