7. (15 points) Let $g$ be the function that is defined for $x > 1$ by

$$g(x) = \int_{3}^{x} \frac{t}{\ln t} \, dt.$$ 

(a) Find $g'(x)$.

$$g'(x) = \frac{x}{\ln(x)}$$ 

(b) On which subinterval(s) of $x > 1$, if any, is $g$ increasing? Briefly explain the reason for your answer.

$g$ is increasing where $g' > 0$, or where $\frac{x}{\ln(x)} > 0$. Thus $g$ is increasing on every subinterval of $x > 1$.

(c) On which subinterval(s) of $x > 1$, if any, is $g$ concave up? Briefly explain the reason for your answer.

$g$ is concave up where $g'$ is increasing or where $g'' > 0$. Now, $g''(x) = \frac{\ln(x) - 1}{(\ln(x))^2}$. Since the denominator is always positive, this expression is zero at $x = e$, positive for $x > e$ where $\ln x > 1$, and negative for $1 < x < e$ where $\ln x - 1 < 0$. Therefore, $g''(x) > 0$ and $g$ is concave up for $x > e$.

(d) Fill in the blanks with one of the words “positive”, “negative”, or “zero”, to make the following sentences true. (Any word may be used more than once. No explanation necessary.)

$g(4)$ is \boxed{positive}.

$g(3)$ is \boxed{zero}.

$g(2)$ is \boxed{negative}.