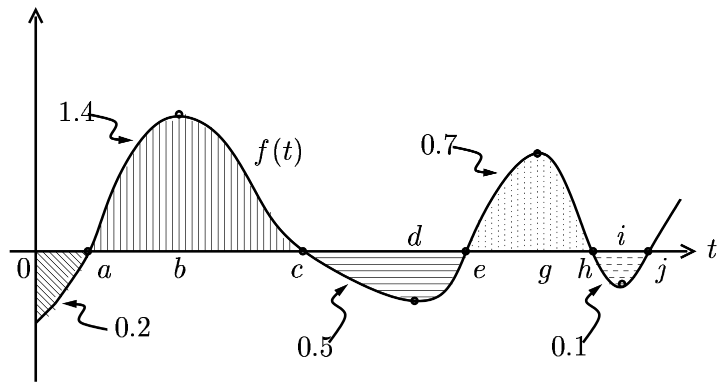
8. (16 pts.) The function $f(t)$ represents the velocity (in meters per second) of a charged particle in a variable electromagnetic field, t seconds after the beginning of an experiment. Positive velocity represents travel away from the positively charged plate used in the experiment. The graph of f is shown below. The areas between the graph of f and the horizontal axis are also indicated.



(a) In the context of the question, briefly explain the meaning of the integral $\int_c^h f(t)\,dt$.

The particle is $\int_c^h f(t)\,dt$ meters farther from the plate at time $t = h$ than it is at time $t = c$.

(b) At which time(s) between $t = 0$ and $t = j$ is the particle furthest from the positively charged plate? How do you know this?

The particle is furthest from the positively charged plate at time $t = h$. This is because the particle is $\int_0^t f(s)\,ds$ meters farther from the plate at time t than it is at time 0. The function g is decreasing for $0 < t < a$, for $c < t < e$, and for $h < t < j$, and it is increasing for $a < t < c$ and $e < t < h$. Thus, g must be largest at either $t = 0$, $t = c$, or $t = h$. However, from the given graph and areas, $g(h) = g(c) - .5 + .7 = g(c) + .2 > g(c) = g(0) - .2 + 1.4 = g(0) + 1.2 > g(0)$. Therefore, for times between $t = 0$ and $t = j$, g is largest at $t = c$.

(c) What is the distance between the position of the particle at time $t = 0$ and its position at time $t = e$. Be sure to show the calculations used to obtain your answer.

0.7 meters. **To find this we calculate $\int_0^a f(t)\,dt + \int_a^c f(t)\,dt + \int_c^e f(t)\,dt$ and note that the graph tells us that $\int_0^a f(t)\,dt = -0.2$, $\int_a^c f(t)\,dt = 1.4$, and $\int_c^e f(t)\,dt = -0.5$. Thus we obtain $(-0.2) + 1.4 + (-0.5) = 0.7$.**

(d) What is the total distance travelled by the particle in the first e seconds? Be sure to show your calculation.

2.1 meters. **To find this we calculate $\int_0^a |f(t)|\,dt + \int_a^c |f(t)|\,dt + \int_c^e |f(t)|\,dt$ and note that the graph tells us that $\int_0^a |f(t)|\,dt = 0.2$, $\int_a^c |f(t)|\,dt = 1.4$, and $\int_c^e |f(t)|\,dt = 0.5$. Thus we obtain $0.2 + 1.4 + 0.5 = 2.1$. (Note that we use $\int_p^q |f(t)|\,dt$ because we are concerned with distance travelled rather than change in displacement.)**