1. (85 points) **Modeling the amount of water in a container.** Consider 3 containers, in which water flows into or out of each container at a different rate. Your job is to determine how much water is in each container at the end of 75 seconds.

   a. If $r(t)$ describes the flow of water into a container with units of milliliters per second (ml/sec), and t is measured in seconds, write a sentence or two explaining what $\int_a^b r(t)dt$ means in this context.

   This definite integral represents the net change in the amount of water (in ml) in the container between $t = a$ seconds and $t = b$ seconds

   b. ***Container 1*:** The initial amount of water in container 1 is 150 milliliters (ml). Water flows into container 1 at a rate $r_1(t)$ ml/sec described by the following data.

Time (sec)	0	25	50	75
$r_1(t)$ (ml/sec)	23	21	6	2

   What is the volume of water at the end of 75 seconds? Describe the method you use, and the accuracy of your method (i.e. exact, over/underestimate, etc.). If you've made assumptions that affect your answer, you should also explain those as well.

   LHS: 150+23(25)+21(25)+6(25)=150+1250=1400 ml. This is an overestimate if we assume that the data is decreasing always.

   RHS: 150+21(25)+6(25)+2(25)=150+725=875 ml. This is an underestimate if we assume that the data is always increasing.

   TRAP=(RHS+LSH)/2=987.5. If students assume data is linear, then trap is exact. Otherwise, we need more info to know if this estimation is over/under.

c. ***Container 2:*** The initial amount of water in container 2 is 150 milliliters (ml). Water flows into container 2 at a rate $r_2(t)$ ml/sec. *An anti-derivative* of $r_2(t)$ is $R_2(t) = \frac{100t}{35}\sin\left(\frac{t}{18}+3\right)$. What is the volume of water at the end of 75 seconds? Describe the method you use, and the accuracy of your method (i.e. exact, over/underestimate, etc.). If you've made assumptions that affect your answer, you should also explain those as well.

150+ $R_2(75) - R_2(0) \approx$ 150+165.63=315.63. This is an exact method. But round-off error may occur, depending on how student writes down answer.

d. ***Container 3:*** The initial amount of water in container 3 is 150 milliliters (ml). Water flows into container 3 at a rate $r_3(t) = \frac{50}{t^2+5t+6} + 10\sin\left(\frac{2\pi}{75}t\right)$ ml/sec. What is the volume of water at the end of 75 seconds? Describe the method you use, and the accuracy of your method (i.e. exact, over/underestimate, etc.). If you've made assumptions that affect your answer, you should also explain those as well.

Find the anti-derivative of $r_3(t)$ and use FTC. This requires partial fractions and a straightforward w-substitution.

$$\int_0^{75} \frac{50}{t^2+5t+6} + 10\sin\left(\frac{2\pi}{75}t\right) dt = \int_0^{75} \frac{50}{t+2} - \frac{50}{t+3} + 10\sin\left(\frac{2\pi}{75}t\right) dt$$

$$= 50\ln|t+2| - 50\ln|t+3| - \frac{750}{2\pi}\cos\left(\frac{2\pi}{75}t\right)\Bigg|_0^{75}$$

$$= \left(50\ln\left(\frac{77}{78}\right) - \frac{750}{2\pi}\cos(2\pi)\right) - \left(50\ln\left(\frac{2}{3}\right) - \frac{750}{2\pi}\cos(2\pi)\right)$$

$$= 50\ln\left(\frac{77}{52}\right) \approx 19.62$$

Add this to 150 to get 169.62. This method is exact.

e. Considering only the first 75 seconds, does container 3 have its maximum amount of water at 75 seconds? Justify your response.

No, $r_3(t)$ becomes negative prior to 75 seconds. This means water is leaving the container when the rate function is negative. I can graph $r_3(t)$ and see that $r_3(t)$ is negative from about 38 seconds to 75 seconds. This means that the container is losing water and cannot have its maximum at 75 seconds.