2. (30 points) The graphs of \( f(t) \), \( h(t) \), and \( k(t) \) are shown below. You may assume that as \( t \to \infty \), the graphs of \( f \), \( h \), and \( k \) continue in a fashion similar to the trend observed in the graph on the right. We define \( g(x) = \int_1^x f(t) \, dt \).

a. What’s \( g'(2) \)?

By FTC, \( g'(x) = 2xf(x^2) \). Thus

\[
g'(2) = 4f(4) \approx 4 \left( \frac{1}{3} \right).
\]

b. What, if anything, could you say about

\[
\lim_{x \to \infty} g(x) = \int_1^\infty f(t) \, dt \text{ if you knew that } h(t) < \frac{1}{t^{\sqrt{2}}} \text{ for } t \geq 6? \text{ Explain your answer.}
\]

It converges. The graph indicates that \( f(t) < h(t) \) for \( t > 6 \). Thus

\[
\int_6^\infty f(t) \, dt < \int_6^\infty h(t) \, dt = \int_6^\infty \frac{1}{3} \, dt.
\]

The last integral converges since \( p = \frac{3}{2} \) and by the comparison test, \( \int_6^\infty f(t) \, dt \) must converge as well. Since \( \int_6^\infty f(t) \, dt = \int_1^6 f(t) \, dt + \int_6^\infty f(t) \, dt \), we’re adding only a finite amount of area and thus

\[
\int_1^\infty f(t) \, dt \text{ converges.}
\]

c. What, if anything, could you say about \( \lim_{x \to \infty} g(x) = \int_1^\infty f(t) \, dt \) if you were to instead assume that \( \int_{100}^\infty k(t) \, dt = 16 \)? Explain your answer.

Inconclusive. The graph indicates that \( f(t) > k(t) \) and thus \( \int_{100}^\infty f(t) \, dt > \int_{100}^\infty k(t) \, dt \). The fact that \( \int_{100}^\infty k(t) \, dt = 16 \) means that the integral converges. But this is not enough to determine whether or not \( \int_{100}^\infty f(t) \, dt \) converges. And this means that we don’t have enough info to determine whether or not \( \int_1^\infty f(t) \, dt \) converges.