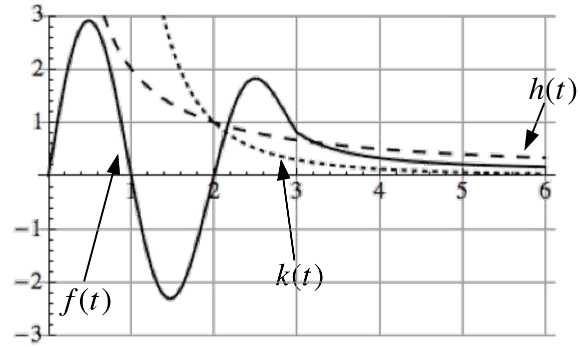
2. (30 points) The graphs of $f(t)$, $h(t)$, and $k(t)$ are shown below. You may assume that as $t \to \infty$, the graphs of *f*, *h*, and *k* continue in a fashion similar to the trend observed in the graph on the right. We define $g(x) = \int_1^{x^2} f(t)\,dt$.



a. What's $g'(2)$?

By FTC, $g'(x) = 2xf(x^2)$. Thus

$g'(2) = 4f(4) \approx 4\left(\frac{1}{3}\right)$.

b. What, if anything, could you say about $\lim_{x\to\infty} g(x) = \int_1^{\infty} f(t)dt$ if you knew that $h(t) < \frac{1}{t\sqrt{t}}$ for $t \geq 6$? Explain your answer.

<u>It converges</u>. The graph indicates that $f(t) < h(t)$ for $t > 6$. Thus $\int_6^{\infty} f(t)dt < \int_6^{\infty} h(t)dt = \int_6^{\infty} \frac{1}{t^{\frac{3}{2}}}\,dt$.

The last integral converges since $p = \frac{3}{2}$ and by the comparison test, $\int_6^{\infty} f(t)dt$ must converge as well. Since $\int_1^{\infty} f(t)dt = \int_1^{6} f(t)dt + \int_6^{\infty} f(t)\,dt$, we're adding only a finite amount of area and thus $\int_1^{\infty} f(t)dt$ converges.

c. What, if anything, could you say about $\lim_{x\to\infty} g(x) = \int_1^{\infty} f(t)dt$ if you were to instead assume that $\int_{100}^{\infty} k(t)dt = 16$? Explain your answer.

<u>Inconclusive.</u> The graph indicates that $f(t) > k(t)$ and thus $\int_{100}^{\infty} f(t)dt > \int_{100}^{\infty} k(t)dt$. The fact that $\int_{100}^{\infty} k(t)dt = 16$ means that the integral converges. But this is not enough to determine whether or not $\int_{100}^{\infty} f(t)dt$ converges. And this means that we don't have enough info to determine whether or not $\int_1^{\infty} f(t)dt$ converges