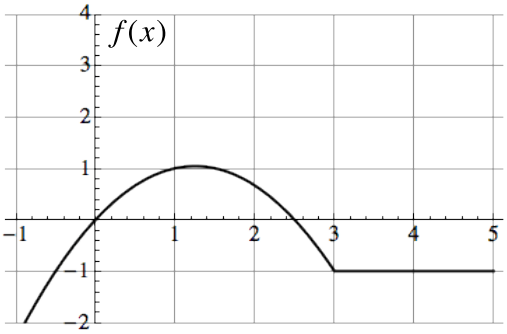
3. (35 points) The graph of $f(x)$ on $[0,4]$ is shown below. You may want to refer to it to answer the following questions.




a. What is $\int_0^4 x f'(x)\,dx$?

Using integration by parts,

$$\int_0^4 f(x)dx \approx xf(x)\Big|_0^4 - \int_0^4 f(x)\,dx$$

Using geometry, I estimate that $\int_0^4 f(x)dx \approx \frac{1}{2}(2.5)(1) - \frac{1}{2}(0.5)(1) - 1(1) = 0$

So $\int_0^4 xf'(x)dx \approx 4\,f(4) - 0\,f(0) - (0) = 4(-1) - 0 = -4$

b. What is $\int_0^2 x f'(x^2)\,dx$?

Using w-substitution,

$$\int_0^2 x f'(x^2)dx = \frac{1}{2}\int_0^4 f'(w)dw = \frac{1}{2}[f(4) - f(0)]$$
$$= \frac{1}{2}[-1-0] = -\frac{1}{2}$$