4. (40 points) The graph of $f^{\prime}(x)$ is shown in the graph below. Given the fact that $f(0)=5$, sketch a rough graph of $f(x)$ on the blank axes provided for the domain [0,10]. You should indicate all critical points, inflection points, and function values (if applicable).



Students should note (graphically or otherwise) that the function is decreasing on $[0,6]$ and increasing on $[6,10]$. There's a local min at $x=6$. The function is concave up on $[0,1] \cup[4,8]$ and concave down on $[1,4] \cup[8,10]$. In addition to $f(0)=5$, we can use FTOC to find other function values. For example, $-11=\int_{0}^{6} f^{\prime}(x) d x=f(6)-f(0)$ and $f(0)=5$ to get $f(6)=-6$. Similarly $f(10)=6$. Other relevant points are inflection points at $(1,3.5),(4,-2)$, and $(8,0)$ (found by estimating area under the graph). Critical points exist at $(6,-6)$ and $(10,6)$ since $f^{\prime}=0$.

