5. **True/False/Explain** (40 points) For each of the following determine whether the statements are true or false. To receive credit you must justify your decision with a relevant sentence or two, calculation or picture explaining your thoughts.

   a. Suppose that a function *h* and its derivative h' are continuous. If $h'(x) < 0$ for all $a \le x \le b$ then every left-hand sum estimate of $\int_a^b h(x)\,dx$ will be an overestimate.

   TRUE $h'(x) < 0$ means that *h* is strictly decreasing. Thus, every left-had sum will be an overestimate.

   b. If $f(x)$ is continuous on $[-5,5]$, then $\int_0^2 |f(x)|\,dx \le \int_0^3 |f(x)|\,dx$

   TRUE. $\int_0^3 |f(x)|\,dx = \int_0^2 |f(x)|\,dx + \int_2^3 |f(x)|\,dx \ge \int_0^2 |f(x)|\,dx$. The last inequality is true since $\int_2^3 |f(x)|\,dx$ must be zero or greater due to the fact $|f(x)|$ is zero or greater.

   c. If $f(x)$ is a positive, continuous function for $x \ge 0$, and if $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)\,dx$ converges.

   FALSE. If $f(x) = \frac{1}{x}$, $\lim_{x\to\infty} f(x) = 0$ but $\int_1^\infty f(x)dx$ diverges (*p*=1).

   d. If $F(x)$ and $G(x)$ are anti-derivatives of a function $f(x)$ that is continuous on $(-\infty,\infty)$, and if $F(5) > G(5)$, then $F(10) > G(10)$.

   TRUE. $F(x) = G(x) + C$ since *F* and *G* are anti-derivatives of the same function. Since $F(5) > G(5)$, the constant *C* must be positive. That means that since $F(10) = G(10) + C$, $F(10) > G(10)$