

1. [11 points] There is a classic result in mathematics, which states that the number of prime numbers less than any number  $x \geq 2$  is approximated by the function  $\text{li}(x) = \int_2^x \frac{dt}{\ln t}$ .
- a. [3 points] Is  $\text{li}(x)$  increasing, decreasing, or neither for  $x \geq 2$ ? Provide justification for your answer.
- b. [3 points] Is  $\text{li}(x)$  concave up, concave down, or neither for  $x \geq 2$ ? Provide justification for your answer.
- c. [5 points] Using Integration by Parts, put  $\text{li}(x)$  into the form

$$\text{li}(x) = f(x) + \int_2^x \frac{dt}{(\ln t)^2}.$$

What is  $f(x)$ ?