3. [8 points] When a spaceship takes off it does not travel in a straight path as it ascends. Instead, it turns slightly east, so that it gains speed by traveling with the rotation of the earth. From mission control's point of view, the spaceship's path appears to follow the curve $y = \sqrt{1+10x^2} - 1$, where y is the height in meters of the spaceship off the ground and x is the horizontal movement in meters from the launch pad. After 20 seconds, the spaceship appears to be 1 kilometer high.

a. [2 points] Determine, to the nearest hundredth of a meter, the horizontal distance the spaceship has traveled from the launch pad at 20 seconds.

Solution: When $y = 1000$ we solve for x:

$$\begin{aligned} 1000 &= \sqrt{1+10x^2} - 1 \\ 1001^2 &= 1 + 10x^2 \\ 1002000 &= 10x^2 \\ x &= \sqrt{100200} \approx 316.54 \text{ meters} \end{aligned}$$

b. [6 points] From mission control's point of view, what is the total distance of the path the spaceship appears to have traveled throughout the first 20 seconds of its trip? Give your answer to the nearest hundredth of a meter, and be sure to show enough work to justify your answer.

Solution: When $y = 0$ we can also see that $x = 0$. Now we use the formula to calculate arc length, noting that $\frac{dy}{dx} = \frac{10x}{\sqrt{1+10x^2}}$, and we get

$$\int_0^{316.543} \sqrt{1 + \frac{100x^2}{1+10x^2}}\, dx \approx 1048.95 \text{ meters},$$

by using the calculator to determine this integral.