4. [12 points] The function $f(x) = \int_0^x 10e^{-t^2} dt$ appears frequently in statistical analysis.

a. [6 points] Without calculating them, order $\int_0^2 f(x)dx$, MID(4), and TRAP(4) from smallest to biggest, where MID(4) and TRAP(4) are approximations for $\int_0^2 f(x)dx$. Show all work to justify your answer.

Solution: Since $f(x)$ is concave down, $\text{TRAP}(4) \le \int_0^2 f(x)dx \le \text{MID}(4)$. We can calculate the concavity of the function $f(x)$ by taking its second derivative:

$$\frac{d^2}{dx^2} f(x) = \frac{d^2}{dx^2} \int_0^x 10e^{-t^2} dt = \frac{d}{dx} 10e^{-x^2} = -20xe^{-x^2} < 0.$$

-OR-

The function $g(x) = 10e^{-x^2}$ is positive and decreasing for $x \ge 0$, so we know $g'(x) < 0$ on this interval. Since $g(x)$ is the derivative of $f(x)$, $g'(x)$ is the second derivative of $f(x)$, and we know that $f(x)$ is concave down, which gives $\text{TRAP}(4) \le \int_0^2 f(x)dx \le \text{MID}(4)$.

b. [2 points] Consider the following table, which evaluates $f(x) = \int_0^x 10e^{-t^2} dt$ for the specified values of x.

x	0	0.5	1	1.5	2
$f(x)$	A	4.613	7.468	8.562	B

What are the values of A and B? Write your answers on the spaces provided, rounding to three decimal places.

A = **0** B = **8.821**

c. [4 points] Using the table provided in part (b) and the answers you found in part (b), calculate LEFT(4) and RIGHT(4) to estimate the integral $\int_0^2 f(x)dx$. Be sure to show enough work to support your answer.

Solution: For four subintervals on the interval $0 \le x \le 2$, we need $\Delta x = 0.5$.

$$\text{LEFT}(4) = 0.5(0 + 4.613 + 7.468 + 8.562) = 10.3215$$

$$\text{RIGHT}(4) = 0.5(4.613 + 7.468 + 8.562 + 8.821) = 14.732$$