4. [12 points] The function $f(x) = \int_0^x 10e^{-t^2} dt$ appears frequently in statistical analysis.

a. [6 points] Without calculating them, order $\int_0^2 f(x)dx$, MID(4), and TRAP(4) from smallest to biggest, where MID(4) and TRAP(4) are approximations for $\int_0^2 f(x)dx$. Show all work to justify your answer.

Solution: Since $f(x)$ is concave down, TRAP(4) $\leq \int_0^2 f(x)dx \leq$ MID(4). We can calculate the concavity of the function $f(x)$ by taking its second derivative:

$$\frac{d^2}{dx^2} f(x) = \frac{d^2}{dx^2} \int_0^x 10e^{-t^2} dt = \frac{d}{dx}10e^{-x^2} = -20xe^{-x^2} < 0.$$  

-OR-

The function $g(x) = 10e^{-x^2}$ is positive and decreasing for $x \geq 0$, so we know $g'(x) < 0$ on this interval. Since $g(x)$ is the derivative of $f(x)$, $g'(x)$ is the second derivative of $f(x)$, and we know that $f(x)$ is concave down, which gives TRAP(4) $\leq \int_0^2 f(x)dx \leq$ MID(4).

b. [2 points] Consider the following table, which evaluates $f(x) = \int_0^x 10e^{-t^2} dt$ for the specified values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>A</td>
<td>4.613</td>
<td>7.468</td>
<td>8.562</td>
<td>B</td>
</tr>
</tbody>
</table>

What are the values of A and B? Write your answers on the spaces provided, rounding to three decimal places.

$$A = \boxed{0} \quad B = \boxed{8.821}$$

c. [4 points] Using the table provided in part (b) and the answers you found in part (b), calculate LEFT(4) and RIGHT(4) to estimate the integral $\int_0^2 f(x)dx$. Be sure to show enough work to support your answer.

Solution: For four subintervals on the interval $0 \leq x \leq 2$, we need $\Delta x = 0.5$.

$$\text{LEFT}(4) = 0.5(0 + 4.613 + 7.468 + 8.562) = 10.3215$$

$$\text{RIGHT}(4) = 0.5(4.613 + 7.468 + 8.562 + 8.821) = 14.732$$