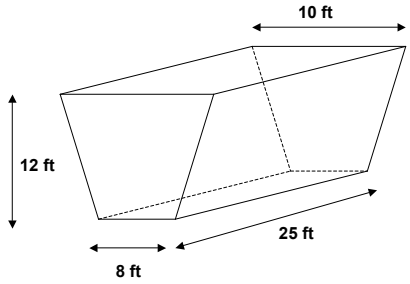
6. [12 points] The dimensions of a large in-ground reservoir are shown in the figure below. (The ends of the reservoir are trapezoids.) The top of the reservoir is at ground level. Currently, water fills the bottom 9 feet of the reservoir. Recall that the density of water is 62.4 lb/ft^3.



a. [6 points] Write an expression that approximates the work done in lifting a horizontal slice of water that is y_i feet below ground level to the ground's surface, given that the depth of the slice is Δy. Include appropriate units in your answer.

Solution: A horizontal slice has length 25 ft, depth Δy ft, and the width is $10 - \frac{1}{6}y_i$ ft, so the volume is volume $= 25(10 - \frac{1}{6}y_i)\Delta y$. The weight of the slice is weight $= 1560(10 - \frac{1}{6}y_i)\Delta y$. The slice must be lifted a distance of y_i feet, so the work on the slice is given by

$$\text{work } = 1560y_i(10 - \frac{1}{6}y_i)\Delta y \text{ ft-lbs.}$$

b. [6 points] How much work is done to pump all of the water currently in the reservoir to the ground's surface? Be sure to include units and show enough work to support your answer.

Solution: Summing up the slices that we found in part (a), we have

$$\text{total work } \approx \sum_{i=1}^{n} 1560y_i(10 - \frac{1}{6}y_i)\Delta y.$$

Letting $n \to \infty$, we have

$$\text{total work } = \int_3^{12} 1560y(10 - \frac{1}{6}y)dy = 905,580 \text{ ft-lbs.}$$