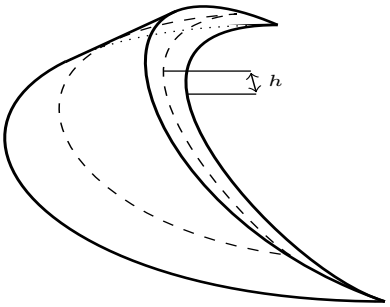
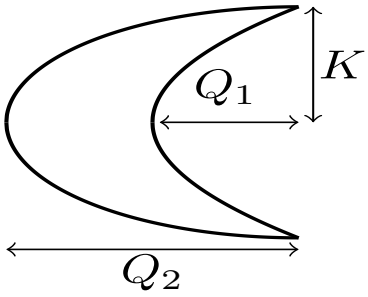
8. [12 points] Sand dunes come in many shapes. *Barchan* dunes, which have the shape shown on the left, are studied extensively by geomorphologists. Horizontal cross-sections of these dunes are crescent-shaped (the dashed line encloses one such cross-section), and can be approximated as the shape on the right. The area of this shape is given by the formula $A_h = K(\frac{\pi}{2}Q_2 - \frac{4}{3}Q_1)$.





You are studying a barchan dune of 10 meters height, for which the values of Q_1, Q_2, and K vary with respect to the height h (in meters) of the cross-section according to the functions $Q_1(h) = 10-h$, $Q_2(h) = 20-2h$, $K(h) = 100-h^2$. The density of sand in the dune is $\delta = 1600$ kilograms per cubic meter.

a. [5 points] Write an expression for the volume of one slice of sand dune h meters above the ground and Δh meters thick.

b. [5 points] Write a definite integral that represents the total mass of sand in the dune. You do not need to evaluate this integral.

c. [2 points] Write an expression (involving integrals) for the height of the center of mass of the sand dune. You do not need to evaluate this integral.