1. [10 points] Indicate if each of the following statements are true or false by circling the correct answer. **Justify your answers.**

a. [2 points] If $F(x)$ is an antiderivative of an even function $f(x)$, then $F(x)$ must also be an even function.

True [False]

Solution: $f(x) = 3x^2$ has $F(x) = x^3 + 1$ as an antiderivative which is not even (not odd either).

b. [2 points] If $G(x)$ is an antiderivative of $g(x)$ and $(G(x) - F(x))' = 0$, then $F(x)$ is an antiderivative of $g(x)$.

[True] False

Solution: $g(x) = G'(x) = F'(x)$ hence $F(x)$ is an antiderivative of $g(x)$.

c. [2 points] Let $f(t) = bt + ct^2$ with $b > 0$ and $c > 0$, then $\text{Left}(n) \leq \int_0^{10} f(t)dt$ for all n.

[True] False

Solution: Since $f'(t) = b + 2ct > 0$ for $t > 0$, then $f(t)$ is increasing on $[0, 10]$ and the left sums yield an underestimate.

d. [2 points] The average of an even function $f(x)$ over the interval $[-a, a]$ is equal to twice its average over the interval $[0, a]$.

True [False]

Solution: Both average are the same. $\frac{1}{2a}\int_{-a}^{a} f(x)dx = \frac{2}{2a}\int_0^a f(x)dx = \frac{1}{a}\int_0^a f(x)dx$.

e. [2 points] The density δ of a circular porcelain dinner plate depends on the distance r from the center of the plate. The relationship between δ and r is shown in the graph below. The center of mass of this plate is located near the edge of the plate.

True [False]

Solution: The center of mass is at the center.