4. [15 points] A patient is given 100 mg of an experimental drug. It has been estimated that the rate $f(t)$ at which his body eliminates the drug is given in the following table. Values of t are in hours after the administration of the drug and $f(t)$ is measured in mg/hour.

t	0	0.5	1.0	1.5	2
$f(t)$	17.3	14.5	12.2	10.3	8.6

Assume $f(t)$ is continuous with no critical points or points of inflection in $0 \leq t \leq 2$. Make sure to include the appropriate units in your answers below.

a. [4 points] Use each left, right, trapezoid and midpoint sums to estimate amount of drug eliminated after 2 hours. When calculating each sum, use the maximum number of subdivisions possible. Show all the terms in each sum.

Solution:

- $Left(4) = .5(17.3 + 14.5 + 12.2 + 10.3) = 27.15$ mg
- $Right(4) = .5(14.5 + 12.2 + 10.3 + 8.6) = 22.8$ mg
- $Trap(4) = \frac{Left(4)+Right(4)}{2} = 24.97$ mg
- $Mid(2) = 1(14.5 + 10.3) = 24.8$ mg.

b. [4 points] Using the computations in a), what is the best overestimate you can find for the amount of drug removed from the patient's body after 2 hours? What is the best underestimate? Justify your answers.

Solution:

$$24.8 \leq \int_0^2 f(t)dt \leq 24.97.$$

$Trap(4)$ yields an overestimate and $Mid(2)$ is an underestimate since $f(t)$ is concave up.

c. [4 points] Using left and right hand sums, how often do we have to measure $f(t)$ in $0 \leq t \leq 2$ to obtain an estimate of the amount of drug eliminated from the patient's body after 2 hours within 0.1 mg of its actual value?

Solution: Since $f(t)$ is decreasing in $[0, 2]$, then we need to subdivide with pieces of length Δt satisfying $|17.3 - 8.6|\Delta t < 0.1$. Hence $\Delta t < .011$ hrs or more than 87 times every hour.

d. [3 points] Find a formula for $g(t)$, the amount of drug (in mg) left in the patient's body after t hours of being administered.

Solution:

$$g(t) = 100 - \int_0^t f(x)dx$$