7. [11 points]
A truck carrying a large tank of paint leaves a garage at 9AM. The tank starts to leak in such a way that x miles from the garage, the density of paint on the road is $e^{-x^2/5000}$ gallons per mile. At 10AM, a cleaning crew leaves from the same garage and follows the path of the truck, scrubbing the paint from the road as it travels until it catches up to the leaking truck. At t hours after 10AM, the leaking truck is $50\ln(t+2)$ miles from the garage, and the cleanup crew is $35t$ miles from the garage. You may use your calculator to evaluate any definite integrals for this problem.

a. [4 points] Calculate the total amount of paint that has leaked from the truck by 11AM.

Solution: The truck at 11 AM is at $50\ln 3 = 54.9306$ miles from the garage. Total amount of paint leaked from the truck is

$$\int_0^{50\ln 3} e^{-x^2/5000}dx = 45.6246 \text{ gallons.}$$

b. [2 points] At time t hours after 10AM, what interval I of the road is still covered in paint? (you may assume that t represents a time before the trucks meet)

Solution: The truck is at $50\ln(t+2)$ miles from the garage and the crew is at $35t$ miles from the garage. $I = [35t, 50\ln(t+2)]$.

c. [3 points] Let $P(t)$ represent the amount of paint in gallons on the road t hours after 10 AM. Find a formula (which may include a definite integral) for $P(t)$.

Solution:

$$P(t) = \int_{35t}^{50\ln(t+2)} e^{-x^2/5000}dx$$

d. [2 points] Calculate $P'(1)$.

Solution:

$$P'(t) = e^{-(50\ln(t+2))^2/5000}\left(\frac{50}{t+2}\right) - 35\left(e^{-(35t)^2/5000}\right)$$

$$P'(1) = e^{-(50\ln(3))^2/5000}\left(\frac{50}{3}\right) - 35\left(e^{-(35)^2/5000}\right) = -18.2795 \text{ gal/hr}$$