8. [12 points] Sand dunes come in many shapes. *Barchan* dunes, which have the shape shown on the left, are studied extensively by geomorphologists. Horizontal cross-sections of these dunes are crescent-shaped (the dashed line encloses one such cross-section), and can be approximated as the shape on the right. The area of this shape is given by the formula \( A_h = K\left(\frac{\pi}{2}Q_2 - \frac{4}{3}Q_1\right) \).

You are studying a barchan dune of 10 meters height, for which the values of \( Q_1, Q_2, \) and \( K \) vary with respect to the height \( h \) (in meters) of the cross-section according to the functions \( Q_1(h) = 10 - h, \) \( Q_2(h) = 20 - 2h, \) \( K(h) = 100 - h^2. \) The density of sand in the dune is \( \delta = 1600 \) kilograms per cubic meter.

a. [5 points] Write an expression for the volume of one slice of sand dune \( h \) meters above the ground and \( \Delta h \) meters thick.

\[
V_{\text{slice}} \approx A_h \Delta h = (100 - h^2) \left[ \frac{\pi}{2} (20 - 2h) - \frac{4}{3} (10 - h) \right] \Delta h.
\]

b. [5 points] Write a definite integral that represents the total mass of sand in the dune. You do not need to evaluate this integral.

\[
M_{\text{dune}} = \int_0^{10} 1600(100 - h^2) \left[ \frac{\pi}{2} (20 - 2h) - \frac{4}{3} (10 - h) \right] dh.
\]

c. [2 points] Write an expression (involving integrals) for the height of the center of mass of the sand dune. You do not need to evaluate this integral.

\[
\bar{h} = \frac{\int_0^{10} 1600h(100 - h^2) \left[ \frac{\pi}{2} (20 - 2h) - \frac{4}{3} (10 - h) \right] dh}{\int_0^{10} (100 - h^2) \left[ \frac{\pi}{2} (20 - 2h) - \frac{4}{3} (10 - h) \right] dh}
\]