7. [8 points] Let $S$ be the solid whose base is the region bounded by the graph of the curve $y=\frac{1}{\sqrt{x(1+a \ln (x))}}$ (for some positive constant $a>0$ ), the $x$-axis, the lines $x=1$ and $x=e$. The cross-sections of $S$ perpendicular to the $x$-axis are squares. Find the exact volume of $S$. Show all your work to receive full credit.


