1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] If $\int_0^2 3f(x) + 1\ dx = 8$, then $\int_0^2 f(x)\ dx = 2$.

True False

Solution:

$$\int_0^2 3f(x)+1\ dx = \int_0^2 3f(x)dx + \int_0^2 1dx = 3\int_0^2 f(x)dx + 2 = 8 \quad \text{then} \quad \int_0^2 f(x)\ dx = 2$$

b. [2 points] If $\int_a^b f(x)dx = 2$ and $\int_a^b g(x)dx = -3$ then $\int_a^b f(x)g(x)dx = -6$.

True **False**

Solution: For example: If $f(x) = 1$ and $g(x) = -\frac{3}{2}x$ with $a = 0$ and $b = 2$, then $\int_a^b f(x)dx = \int_0^2 dx = 2$ and $\int_a^b g(x)dx = \int_0^2 -\frac{3}{2}xdx = -3$.
But $\int_a^b f(x)g(x)dx = \int_0^2 -\frac{3}{2}xdx = -3 \neq -6$.

c. [2 points] If $f(x) = \int_{-2x}^{0} \sqrt{1+t^4}dt$ then $f(x)$ is increasing.

True False

Solution: Since $f'(x) = -\sqrt{1+(-2x)^4}(-2) = 2\sqrt{1+16x^4} > 0$, then $f(x)$ is increasing.

d. [2 points] If $\int_0^1 f(x)dx \leq \int_0^1 g(x)dx$ then $f(x) \leq g(x)$ for $0 \leq x \leq 1$.

True **False**

Solution: For example: $f(x) = 1 - x$ and $g(x) = 2x$, then $\int_0^1 f(x)dx = \int_0^1 1 - xdx = \frac{1}{2}$ and $\int_0^1 g(x)dx = \int_0^1 2xdx = 2$. But $f(0) = 1 \geq g(0) = 0$.

e. [2 points] If $g(x)$ is odd and $\int_1^3 g(x)dx = 2$, then $\int_{-3}^{1} g(x)dx = -2$.

True False

Solution: Since $g(x)$ is odd, then $\int_{-1}^{1} g(x)dx = 0$ and $\int_{-b}^{-a} g(x)dx = -\int_a^b g(x)dx$. Hence $\int_{-3}^{1} g(x)dx = \int_{-3}^{-1} g(x)dx + \int_{-1}^{1} g(x)dx = \int_{-3}^{-1} g(x)dx = -\int_1^3 g(x)dx = -2$.

f. [2 points] If $f(t)$ is measured in dollars per year, and t is measured in years, then $\int_a^b f(t)dt$ is measured in dollars per years squared.

True **False**

Solution: The units for $\int_a^b f(t)dt$ are dollars (dollars per year (units for $f(t)$) times year (units for t)).