1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] If \( \int_0^2 3f(x) + 1 \, dx = 8 \), then \( \int_0^2 f(x) \, dx = 2 \).

\[ \text{Solution:} \]
\[ \int_0^2 3f(x) + 1 \, dx = \int_0^2 3f(x) \, dx + \int_0^2 1 \, dx = 3 \int_0^2 f(x) \, dx + 2 = 8 \] then \( \int_0^2 f(x) \, dx = 2 \)

b. [2 points] If \( \int_a^b f(x) \, dx = 2 \) and \( \int_a^b g(x) \, dx = -3 \) then \( \int_a^b f(x)g(x) \, dx = -6 \).

\[ \text{Solution:} \text{ For example: If } f(x) = 1 \text{ and } g(x) = -\frac{3}{2}x \text{ with } a = 0 \text{ and } b = 2, \text{ then } \int_a^b f(x) \, dx = \int_0^2 dx = 2 \text{ and } \int_a^b g(x) \, dx = \int_0^2 -\frac{3}{2}x \, dx = -3. \]
But \( \int_a^b f(x)g(x) \, dx = \int_0^2 -\frac{3}{2}x \, dx = -3 \neq -6 \).

c. [2 points] If \( f(x) = \int_{-2}^x \sqrt{1 + t^2} \, dt \) then \( f(x) \) is increasing.

\[ \text{Solution:} \text{ Since } f'(x) = -\sqrt{1 + (-2x)^4}(-2) = 2\sqrt{1 + 16x^4} > 0, \text{ then } f(x) \text{ is increasing.} \]

d. [2 points] If \( \int_0^1 f(x) \, dx \leq \int_0^1 g(x) \, dx \) then \( f(x) \leq g(x) \) for \( 0 \leq x \leq 1 \).

\[ \text{Solution:} \text{ For example: } f(x) = 1 - x \text{ and } g(x) = 2x, \text{ then } \int_0^1 f(x) \, dx = \int_0^1 1 - x \, dx = \frac{1}{2} \]
and \( \int_0^1 g(x) \, dx = \int_0^1 2x \, dx = 2. \text{ But } f(0) = 1 \geq g(0) = 0. \)

e. [2 points] If \( g(x) \) is odd and \( \int_1^3 g(x) \, dx = 2 \), then \( \int_{-1}^1 g(x) \, dx = -2 \).

\[ \text{Solution:} \text{ Since } g(x) \text{ is odd, then } \int_{-1}^1 g(x) \, dx = 0 \text{ and } \int_{-a}^a g(x) \, dx = - \int_a^b g(x) \, dx. \text{ Hence } \int_{-3}^3 g(x) \, dx = \int_{-1}^-1 g(x) \, dx + \int_{-1}^1 g(x) \, dx = \int_{-1}^-1 g(x) \, dx = - \int_1^3 g(x) \, dx = -2. \]

f. [2 points] If \( f(t) \) is measured in dollars per year, and \( t \) is measured in years, then \( \int_a^b f(t) \, dt \) is measured in dollars per years squared.

\[ \text{Solution:} \text{ The units for } \int_a^b f(t) \, dt \text{ are dollars (dollars per year (units for } f(t)) \text{ times year (units for } t)). \]