3. [12 points] A boat travels in a straight line toward an island d km away. The velocity $v(t)$ (toward the island is positive velocity) in km/hr, t hours after departing from its starting position. The velocity $v(t)$ during the first three hours is recorded at half hour intervals, and is given in the table below:

t	0	0.5	1	1.5	2	2.5	3
$v(t)$	50	48	44	38	30	20	8

a. [8 points] Find an estimate for how far the boat is from the starting point after 3 hours using the four approximations LEFT, RIGHT, MID and TRAP. Use the maximum number of subintervals possible. Write each sum, and justify whether the sum is an underestimate or an overestimate. Assume the velocity is always decreasing and has no inflection points. **Circle your answers**.

Solution:

$$\text{LEFT}(6) = \frac{1}{2}(50+48+44+38+30+20) = 115 \qquad \text{overestimate.}$$

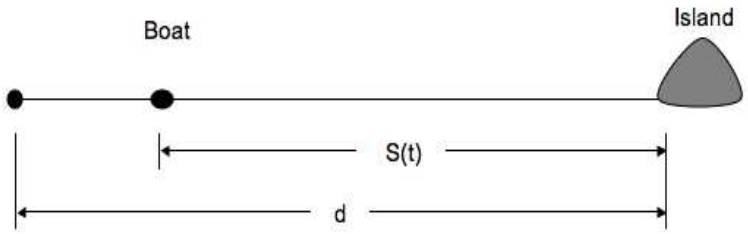
$$\text{RIGHT}(6) = \frac{1}{2}(48+44+38+30+20+8) = 94 \qquad \text{underestimate.}$$

$$\text{TRAP}(6) = \frac{1}{2}(115+94) = 104.5 \qquad \text{underestimate.}$$

$$\text{MID}(3) = (48+38+20) = 106 \qquad \text{overestimate.}$$

since $v(t)$ is decreasing and concave down.

b. [4 points] If it takes the boat 5 hours to reach the island, write an expression involving integrals for the distance $S(t)$ between the island and the boat for $0 \le t \le 5$.



Solution:

$$S(t) = d - \int_0^t v(x)dx.$$