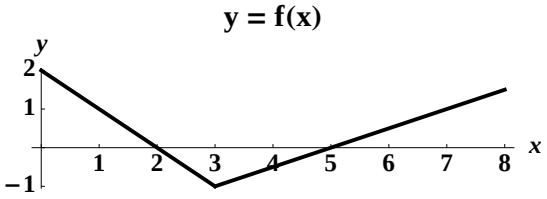
5. [12 points] The graph of $f(x)$ and a table of values for the continuous functions $g(x)$ and $h(x)$ are given below. The function $h(x)$ is an antiderivative of $g(x)$.



x	0	1	2	3	4
$g(x)$	1	3	5	7	9
$h(x)$	-3	-1	3	9	17

Compute the **exact** value of each of the following expressions:

a. [1 point] $\int_0^7 |f(x)|dx$

Solution: $\int_0^7 |f(x)|dx = 2 + \frac{3}{2} + 1 = \frac{9}{2}$.

b. [4 points] $\int_1^{e^2} \frac{f(\ln x)}{x} dx$

Solution: If $u = \ln x$, then

$$\int_1^{e^2} \frac{f(\ln x)}{x} dx = \int_0^2 f(u)du = 2$$

c. [7 points] Find $\int_1^2 xg'(2x)dx$

Solution: If $w = 2x$, then

$$\int_1^2 xg'(2x)dx = \frac{1}{2}\int_2^4 \frac{w}{2} g'(w)dw = \frac{1}{4}\int_2^4 wg'(w)dw.$$

Integration by parts with $u = w$ and $v' = g'(w)$ yields

$$\begin{aligned}\int_2^4 wg'(w)dw &= wg(w)\,\big|_2^4 - \int_2^4 g(w)dw \\ &= 4g(4) - 2g(2) - (h(4) - h(2)) \\ &= 4(9) - 2(5) - (17 - 3) = 12.\end{aligned}$$

$$\int_1^2 xg'(2x)dx = \frac{1}{4}\int_2^4 wg'(w)dw = \frac{1}{4}(12) = 3.$$