6. [11 points] The lateral faces of a tank are determined by the curve $y = 1 - x^4$ and the $x$-axis (where $x$ and $y$ are measured in meters). The length of the tank is 10 meters. Be sure to include units in your answers.

a. [5 points] The tank is filled with water to a height of one half a meter. If the density of water is 1,000 kg/m$^3$, write an expression that approximates the mass of one slice of water $y$ meters above the ground and $\Delta y$ meters thick.

**Solution:** Mass is density times volume. The density is constant, and the volume of one slice at height $y$ with thickness $\Delta y$ is

$$20(1 - y)^{1/4} \Delta y \text{ m}^3.$$ 

Therefore, the mass of the slice is

$$20,000(1 - y)^{1/4} \Delta y \text{ kg.}$$

b. [2 points] Write a definite integral that represents the total mass of water in the tank.

**Solution:** We add up all of the slices from 0 to .5:

$$\int_{0}^{.5} 20,000(1 - y)^{1/4} \, dy \text{ kg.}$$

c. [4 points] Write a definite integral that represents the amount of work required to pump the water to the top of the tank.

**Solution:** Work is force times distance. The force on a slice at height $y$ with thickness $\Delta y$ (or $dy$) is 9.8 times the mass, which we computed above. This slice travels a distance $(1 - y)$ under this force. Therefore the work done on one slice is

$$9.8 \cdot 20,000(1 - y)^{1/4} \cdot (1 - y) \Delta y \text{ N.}$$

So we add up all of those slices to get

$$\int_{0}^{.5} 9.8 \cdot 20,000(1 - y)^{1/4} \cdot (1 - y) \, dy \text{ J or N \cdot m.}$$