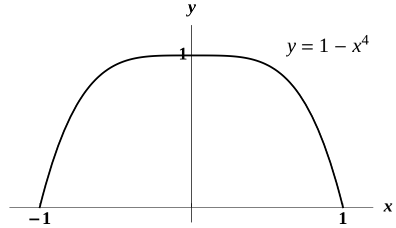
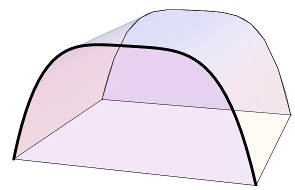
6. [11 points] The lateral faces of a tank are determined by the curve $y = 1 - x^4$ and the x-axis (where x and y are measured in meters). The length of the tank is 10 meters. Be sure to include units in your answers.





a. [5 points] The tank is filled with water to a height of one half a meter. If the density of water is $1,000$ kg/m^3, write an expression that approximates the mass of one slice of water y meters above the ground and Δy meters thick.

Solution: Mass is density times volume. The density is constant, and the volume of one slice at height y with thickness Δy is

$$20(1-y)^{1/4}\Delta y \quad \text{m}^3.$$

Therefore, the mass of the slice is

$$20,000(1-y)^{1/4}\Delta y \quad \text{kg}.$$

b. [2 points] Write a definite integral that represents the total mass of water in the tank.

Solution: We add up all of the slices from 0 to .5:

$$\int_0^{.5} 20,000(1-y)^{1/4}\,dy \quad \text{kg}.$$

c. [4 points] Write a definite integral that represents the amount of work required to pump the water to the top of the tank.

Solution: Work is force times distance. The force on a slice at height y with thickness Δy (or dy) is 9.8 times the mass, which we computed above. This slice travels a distance $(1-y)$ under this force. Therefore the work done on one slice is

$$9.8 \cdot 20,000(1-y)^{1/4} \cdot (1-y)\Delta y \ \ \text{N}.$$

So we add up all of those slices to get

$$\int_0^{.5} 9.8 \cdot 20,000(1-y)^{1/4} \cdot (1-y)\ dy \ \ \text{J or N} \cdot \text{m}.$$