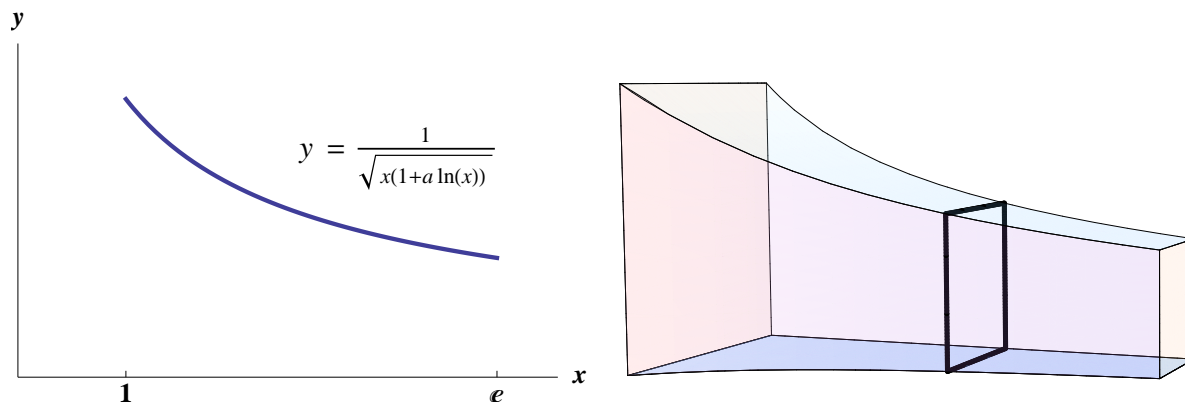**7.** [8 points]   Let S be the solid whose base is the region bounded by the graph of the curve $y = \dfrac{1}{\sqrt{x(1+a\ln(x))}}$ (for some positive constant $a > 0$), the x-axis, the lines $x = 1$ and $x = e$. The cross-sections of S perpendicular to the x-axis are squares. Find the exact volume of S. Show all your work to receive full credit.

Solution: As the cross-section of each slice is a square with sidelength y, the volume of one slice is $y^2 \Delta x$, Therefore the total volume is

$$
\int_1^e \left(\frac{1}{\sqrt{x(1 + a\ln x)}} \right)^2 dx = \int_1^e \frac{1}{x(1 + a\ln x)}\, dx
$$
$$
= \frac{1}{a} \int_1^{1+a} \frac{1}{u}\, du
$$
$$
= \frac{1}{a} \ln|1 + a|.
$$

The second line comes from the u-substitution $u = 1 + a\ln x$.