**9**. [11 points] In the following problems show all your work to receive full credit.

**a**. [7 points] The population of an invasive aquatic plant in a circular lagoon has density given by $\delta(r) = 20(1 - e^{-r^2})$ kg/m$^2$, where $r$ is the distance in meters from its center. The lagoon has radius $R$ meters. Find the exact amount of plants living at the lake.

*Solution:* Since density is a function of radius, we will slice the biomass into rings with thickness $\Delta r$. The mass of one ring is approximately

$$\delta(r) \cdot 2\pi r \Delta r.$$

So now we add up all the slices, and we get total mass

$$\begin{aligned}\int_0^R 20\left(1 - e^{-r^2}\right) \cdot 2\pi r \, dr &= 20\pi \int_0^R 2r\left(1 - e^{-r^2}\right) \, dr \\ &= 20\pi \int_0^{R^2} \left(1 - e^{-u}\right) \, du \\ &= 20\pi \left(u + e^{-u}\right)\Big|_0^{R^2} \\ &= 20\pi(R^2 + e^{-R^2} - 1).\end{aligned}$$

**b**. [4 points] Let

$$F(x) = \int_0^x \sqrt{e^{2t} - 1} dt.$$

Find the exact value of the length of the curve on $0 \le x \le 1$.

*Solution:* The formula for length of an arc is

$$L = \int_0^1 \sqrt{1 + (F'(x))^2} \, dx.$$

By the second FTC, $F'(x) = \sqrt{e^{2x} - 1}$. Therefore the length of the arc is

$$\begin{aligned}L = \int_0^1 \sqrt{1 + (\sqrt{e^{2x} - 1})^2} \, dx &= \int_0^1 \sqrt{1 + (e^{2x} - 1)} \, dx \\ &= \int_0^1 e^x \, dx \\ &= e - 1.\end{aligned}$$