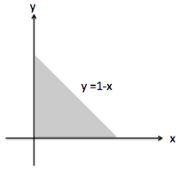
1. [10 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let ($\bar{x}$,$\bar{y}$) be the center of mass of the metal plate bounded by the line $y = 1-x$, the x-axis and the y-axis for $0 \leq x \leq 1$.



If the plate has uniform mass density, then $\bar{x} = \bar{y}$.

[True] False

Solution: Since the plate has uniform density and it is symmetric about the line $y = x$, then $\bar{y} = \bar{x}$. It also follows using the formulas for the coordinates of the center of mass

$$\bar{x} = \frac{\int_0^1 \delta x(1-x)dx}{\int_0^1 \delta(1-x)dx} = \frac{\int_0^1 \delta y(1-y)dy}{\int_0^1 \delta(1-y)dy} = \bar{y}.$$

b. [2 points] The function $F(x) = \displaystyle\int_1^{x^2} \sin(e^t)dt$ is an even function.

[True] False

Solution: The function $F(x)$ is even if it satisfies $F(-x) = F(x)$. Since

$$F(-x) = \int_1^{(-x)^2} \sin(e^t)dt = \int_1^{x^2} \sin(e^t)dt = F(x).$$

then $F(x)$ is even.

c. [2 points] Let $h(x)$ be an antiderivative of $g(x)$. If $g(x)$ is measured in kg and x in inches, then the units for $h(x)$ are kg per inch.

True [False]

Solution: The second fundamental theorem of calculus says that $h(x) = \displaystyle\int_a^x g(t)dt$ for some contant a. The units of $g(x)$ and x are kg and inches respectively, then the units of $h(x)$ are kg $\cdot$ inches.

d. [2 points] The function $R(t) = \displaystyle\int_t^{1-t} e^{x^3}dx$ is decreasing for all values of t.

[True] False

Solution:

$$R'(t) = -e^{(1-t)^3} - e^{t^3} < 0 \text{ for all values of } t.$$

Hence $R(t)$ is decreasing.

e. [2 points] The length of the curve $y = x^2$ from $x = 0$ to $x = 2$ is smaller than 4.

True **[False]**

Solution: The length L of the curve $y = x^2$ from $x = 0$ to $x = 2$ is given by

$$L = \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^2 \sqrt{1 + (2x)^2} dx.$$

Using your calculator you can find $L \approx 4.64$. Or, you can argue without using your calculator. The length L of the curve $y = x^2$ from $x = 0$ to $x = 2$ is larger than the length L_1 of the line connecting the points $(0, 0)$ and $(2, 4)$. Since $L_1 = \sqrt{(2-0)^2 + (4-0)^2} > \sqrt{4^2} = 4$. Hence $L > L_1 > 4$.