2. [14 points] Let $f(x)$ be a continuous function on $0 \le x \le 2$. The values of $f(x)$ are shown below

x	0	0.5	1	1.5	2
$f(x)$	-3	-2	1	3	4

a. [2 points] Use the left-hand sum with four subintervals to approximate the value of $\int_0^2 f(x)dx$. Show all the terms in the sum, and then calculate the numerical value.

Solution:

$$\text{Left}(4) = \frac{1}{2}(-3 - 2 + 1 + 3) = -\frac{1}{2}.$$

b. [2 points] Assume that $f(x)$ has no critical points for $0 \le x \le 2$. Is your estimate in (**a**) guaranteed to be an underestimate or overestimate of $\int_0^2 f(x)dx$, or there is not enough information to decide? Justify.

Solution: Underestimate, because $f(x)$ is increasing.

c. [2 points] Use the trapezoid rule with four subintervals to approximate the value of $\int_0^2 f(x)dx$. Show all the terms in the sum, and then calculate the numerical value.

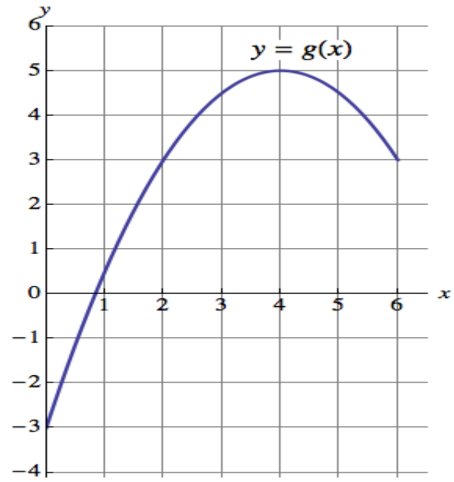
Solution:

$$\frac{\frac{1}{2}(-3 - 2 + 1 + 3) + \frac{1}{2}(-2 + 1 + 3 + 4)}{2} = \frac{-\frac{1}{2} + 3}{2} = 1.25.$$

d. [2 points] Given the data for $f(x)$, is your estimate in (**c**) guaranteed to be an underestimate or overestimate of $\int_0^2 f(x)dx$, or there is not enough information to decide? Justify.

Solution: There's not enough information to decide, because the data shows that f is not always concave up or always concave down.

e. [2 points] Consider the function $g(x)$ whose graph is shown below



Use the midpoint rule with three subintervals to approximate the value of $\int_0^6 g(x)dx$. Show all the terms in the sum, and then calculate the numerical value.

Solution: Mid(3) $= 2(.5 + 4.5 + 4.5) = 19.$

f. [2 points] Use the right-hand sum with three subintervals to approximate the value of $\int_1^3 e^{\sqrt{t}}dt$. Show all the terms in the sum, and then calculate the numerical value.

Solution:

$$\text{Right}(3) = \frac{2}{3}(e^{\sqrt{5/3}} + e^{\sqrt{7/3}} + e^{\sqrt{3}}) \approx \frac{2}{3}(3.636 + 4.606 + 5.652) \approx 9.262$$

g. [2 points] Is your estimate in (**f**) guaranteed to be an underestimate or overestimate of $\int_1^3 e^{\sqrt{t}}dt$, or there is not enough information to decide? Justify.

Solution: We calculate

$$\frac{d}{dt}e^{\sqrt{t}} = \frac{1}{2}\left(\frac{1}{\sqrt{t}}e^{\sqrt{t}}\right) > 0,$$

so $e^{\sqrt{t}}$ is increasing. Therefore Right(3) is an overestimate.