3. [14 points] Consider functions $f(x)$ and $g(x)$ satisfying:

(i) $g(x)$ is an odd function.

(ii) $\int_2^7 g(x)dx = 3.$

(iii) $\int_2^7 f(x)dx = 17.$

(iv) $f(2) = 1.$

(v) $\int_1^6 f'(x)dx = 12.$

(vi) $\int_2^7 f'(x)dx = 3.$

Compute the value of the following quantities. If it is impossible to determine their value with the information provided above, write "NI" (not enough information).

a. [2 points] $\int_{-2}^7 g(x)\ dx =$ _______

Solution: 3, using i and vi.

b. [2 points] $\int_2^7 (f(x) - 8g(x))\ dx =$ _______

Solution: -7, using ii and iii.

c. [2 points] $f(7) =$ _______

Solution: 4, using the Fundamental Theorem of Calculus with iv and vi.

d. [2 points] $\int_1^6 f'(x+1)\ dx =$ _______

Solution: We use u substitution, $u = x + 1$. Making sure to change the limits of integration, we get $\int_2^7 f'(u)du = 3.$

e. [3 points] $\int_2^7 xf'(x)\ dx =$ ________

Solution: We integrate by parts with $u = x, dv = f'$.

$$\int_2^7 xf'(x)dx = xf(x)|_2^7 - \int_2^7 f(x)dx = (7f(7) - 2f(2)) - 17 = 28 - 2 - 17 = 9.$$

f. [3 points] $\int_2^3 xf(x^2 - 2)\ dx =$ ________

Solution: We use u substitution $u = x^2 - 2, du = 2xdx$. We get $\frac{1}{2}\int_2^7 f(u)du = \frac{17}{2} = 8.5$.