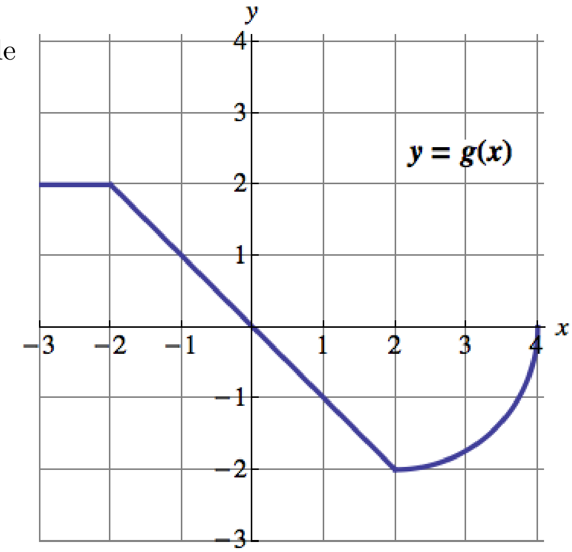
5. [15 points]

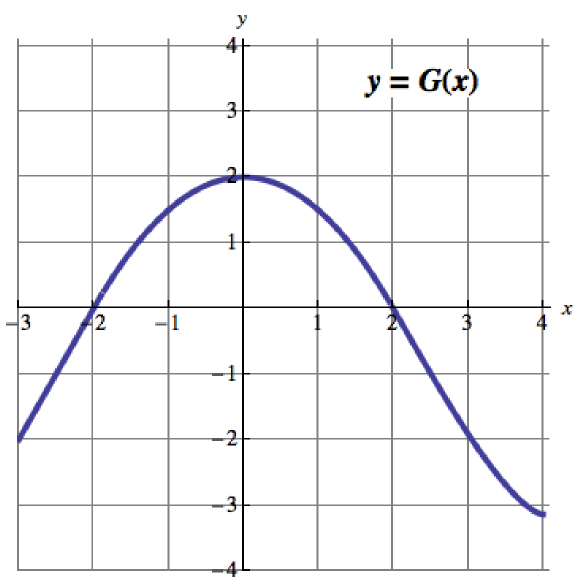
a. [11 points] Let $G(x) = \int_{-2}^{x} g(t)dt$ where the graph of the function $g(x)$ is shown below. The graph of $g(x)$ is a quarter of a circle for $2 \le x \le 4$.

Fill in the indicated values of $G(x)$ in the table below.

x	-3	-2	0	2	4
$G(x)$					



Draw the graph of $G(x)$ for $-3 \le x \le 4$. Make sure your graph indicates the regions where the function $G(x)$ is increasing, decreasing, concave up or concave down, and appropriately reflects the critical points of $G(x)$.



Solution:

x	-3	-2	0	2	4
$G(x)$	-2	0	2	0	$-\pi$

b. [4 points] Consider the function

$$f(x) = \begin{cases} -x & \text{for } x \le 0 \\ x^2 & \text{for } 0 < x. \end{cases}$$

Let $F(x)$ be an antiderivative of $f(x)$ with $F(-2) = 0$. Find a formula for $F(x)$. Your answer should not include any integrals.

$$F(x) = \begin{cases} \rule{5cm}{0.4pt} & \text{for } x \le 0 \\ \\ \rule{5cm}{0.4pt} & \text{for } 0 < x. \end{cases}$$

Solution: We know that $F(x) = \int_{-2}^{x} f(t)dt$. For $x \le 0$, this tells us that $F(x) = \int_{-2}^{x}(-t)dt = \frac{-x^2}{2} + 2^2/2 = -x^2/2 + 2$.
For $x > 0$, we have

$$\int_{-2}^{x} f(t)dt = \int_{-2}^{0}(-t)dt + \int_{0}^{x} t^2 dt = 2 + x^3/3.$$

$$F(x) = \begin{cases} -x^2/2 + 2 & \text{for } x \le 0 \\ \\ 2 + x^3/3 & \text{for } 0 < x. \end{cases}$$