5. [15 points]

a. [11 points] Let \( G(x) = \int_{-2}^{x} g(t) \, dt \) where the graph of the function \( g(x) \) is shown below. The graph of \( g(x) \) is a quarter of a circle for \( 2 \leq x \leq 4 \).

Fill in the indicated values of \( G(x) \) in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(0)</th>
<th>(2)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw the graph of \( G(x) \) for \( -3 \leq x \leq 4 \). Make sure your graph indicates the regions where the function \( G(x) \) is increasing, decreasing, concave up or concave down, and appropriately reflects the critical points of \( G(x) \).

**Solution:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(0)</th>
<th>(2)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G(x) )</td>
<td>(-2)</td>
<td>(0)</td>
<td>(2)</td>
<td>(0)</td>
<td>(-\pi)</td>
</tr>
</tbody>
</table>
b. [4 points] Consider the function

\[ f(x) = \begin{cases} 
-x & \text{for } x \leq 0 \\
 x^2 & \text{for } 0 < x.
\end{cases} \]

Let \( F(x) \) be an antiderivative of \( f(x) \) with \( F(-2) = 0 \). Find a formula for \( F(x) \). Your answer should not include any integrals.

\[ F(x) = \begin{cases} 
-x^2/2 + 2 & \text{for } x \leq 0 \\
2 + x^3/3 & \text{for } 0 < x.
\end{cases} \]

**Solution:** We know that \( F(x) = \int_{-2}^{x} f(t) dt \). For \( x \leq 0 \), this tells us that \( F(x) = \int_{-2}^{x} (-t) dt = \frac{-t^2}{2} + \frac{2^2}{2} = \frac{-x^2}{2} + 2 \).

For \( x > 0 \), we have

\[
\int_{-2}^{x} f(t) dt = \int_{-2}^{0} (-t) dt + \int_{0}^{x} t^2 dt = 2 + x^3/3.
\]

Thus, \( F(x) = \begin{cases}
-x^2/2 + 2 & \text{for } x \leq 0 \\
2 + x^3/3 & \text{for } 0 < x.
\end{cases} \)