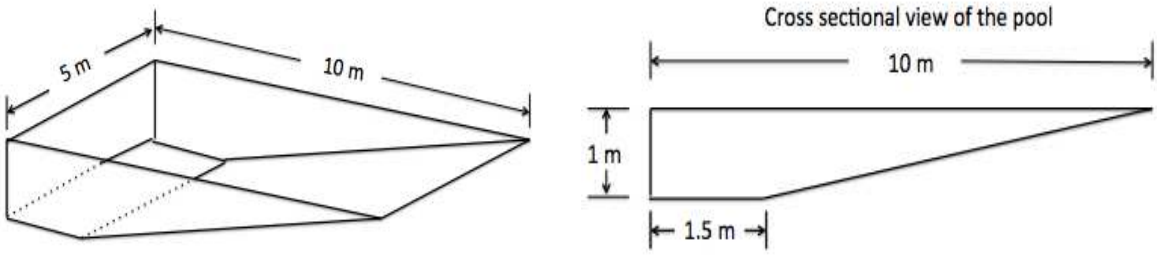
6. [11 points] A swimming pool 10 m long and 5 m wide has varying depth. Its maximum depth is 1 m as shown in the picture below



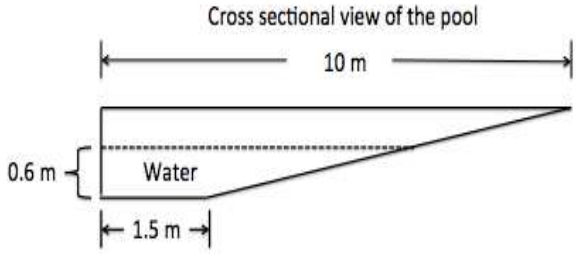
The swimming pool has water up to a level of maximum depth of 0.6 m. The density of water is 1000 kg per m^3. Use $g = 9.8$ m/s^2 for the acceleration due to gravity.

a. [9 points] Write an expression that approximates the work done in lifting a horizontal slice of water with thickness Δy meters, that is at a distance of y meters above the bottom, to the top of the swimming pool.

Solution: First we must find a formula for the length of the swimming pool at depth for a given height above the bottom. Let's call this function $l(y)$. We know that $l(0) = 1.5$ and $l(1) = 10$. Since $l(y)$ is a linear function, this tells us that $l(y) = 8.5y + 1.5$.
The volume of such a slice is $\Delta y(8.5y + 1.5) \cdot 5$. Multiplying by 1000 kg/m^3 and 9.8 m/s^2 gives us the weight of the water in Newtowns. The amount the water needs to be lifted is $(1 - y)$. We therefore get:

$$W_{slice} \approx 1000 \cdot 9.8 \cdot (8.5y + 1.5) \cdot 5 \cdot (1 - y)\Delta y.$$

b. [2 points] Write a definite integral that computes the work required to pump all the water to the top of the pool.



Solution: Work $= \int_0^{0.6} 1000 \cdot 9.8 \cdot 5(8.5y + 1.5)(1 - y)dy$ Joules.