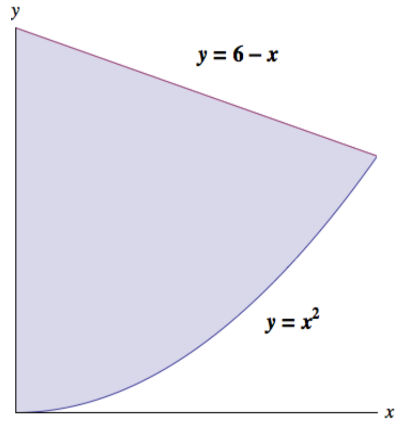
7. [8 points] Let S be the solid whose base is the region bounded by the curves $y = x^2$, $y = 6 - x$ and $x = 0$ and whose cross sections **parallel** to the x-axis are squares. Find a formula involving definite integrals that computes the volume of S.



Solution: First we solve for where the two curves intersect. $6-x = x^2$ implies $0 = x^2+x-6 = (x+3)(x-2)$, so $x = 2$, which implies $y = 4$. We have to split the problem into two cases, one when $0 \le y \le 4$, and one when $4 \le y \le 6$. We will choose thin horizontal slices, and integrate in terms of y, so we need to solve for x in terms of y: $x = \sqrt{y}$ and $x = 6 - y$ are our two curves.

In the case of $0 \le y \le 4$, a thin slice has volume $V_{slice} \approx (\sqrt{y})^2 \Delta y$. In the second case, $4 \le y \le 6$, a thin slice has volume $V_{slice} \approx (6-y)^2 \Delta y$. Hence the total volume of the solid is given by

$$V = \int_0^4 y dy + \int_4^6 (6-y)^2 dy.$$