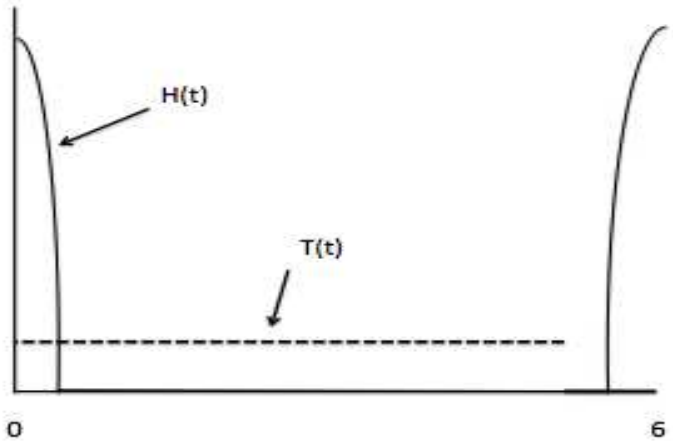
8. [11 points] A tortoise and a hare decide to race. They decide to race a straight 5 kilometer course. The race starts at 12pm. The hare is much faster than the tortoise, so he's confident that he'll win. The hare runs very fast for 30 minutes, getting to what it knows is the half-way point. The hare is tired (it had been studying for exams the night before), so it decides to take a nap. It falls asleep for 5 hours, wakes up, discovers that (now that it's 5:30) it's dark, and runs to the finish line, arriving at 6pm. When it gets there, it's surprised to see the tortoise is already there. "I hope you enjoyed your nap! I've been here for an hour, since 5 o'clock!" the tortoise says. "Steady and slow is the way to go: I kept going the same speed the whole time."

Let $H(t)$ be the hare's velocity and $T(t)$ be the tortoise's velocity, in km per hour, where t is measured in hours after 12pm.



Let

$$R(t) = \int_0^t H(s)ds - \int_0^t T(s)ds.$$

a. [1 point] At times when $R(t) > 0$, who is winning the race?

Solution: The hare

b. [2 points] What is the practical interpretation of the function $|R(t)|$? Include units.

Solution: $|R(t)|$ is the distance in km between the tortoise and the hare t hours after 12pm.

c. [3 points] For what values of $0 \leq t \leq 6$, does $R(t) = 0$?

Solution: $t = 0, t = 2.5, t = 6$.

d. [2 points] For what values of $0 \leq t \leq 6$ is the function $\dfrac{dR}{dt} < 0$?

Solution: $0.5 < t < 5$.

e. [3 points] Write down a definite integral that represents the hare's average velocity from 12 to 12:30. What is the value of the hare's average velocity during this time?

Solution: $\frac{1}{.5}\int_0^{1/2} H(s)ds$. We know that $\int_0^{1/2} H(s)ds = 2.5$, because the Hare has gotten halfway by 12:30. Therefore, the average velocity is 5 km/hr.